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Nonexistence of global solutions of the mixed problem for a system of nonlinear wave equations with q-Laplacian operators

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Abstract. *We study the mixed problem for the system of nonlinear q-Laplacian wave equations and investigate the nonexistence of the global solutions.*

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1 Introduction

In this paper we consider the nonlinear initial-boundary value problem

$$\begin{cases} u_{1tt} - \Delta_q u_1 + (-\Delta)^\alpha u_{1t} = f_1(u_1, u_2), \\ u_{2tt} - \Delta_q u_2 + (-\Delta)^\alpha u_{2t} = f_2(u_1, u_2), \end{cases} \tag{1.1}$$

$$u_j(t, x) = 0, \quad t > 0, \quad x \in \partial\Omega, \quad j = 1, 2, \tag{1.2}$$

$$u_j(0, x) = \varphi_j(x), \;\; u_{jt}(0, x) = \psi_j(x), \quad x \in \Omega, \quad j = 1, 2. \tag{1.3}$$

Here Ω is a bounded domain in $\mathbb{R}^n$, $n \geq 1$ with smooth boundary $\partial\Omega$, $t > 0$, $x \in \Omega$, $0 < \alpha \leq 1$,

$$f_1(u_1, u_2) = |u_1 + u_2|^{2\rho}(u_1 + u_2) + |u_1|^{\rho-1}|u_2|^{\rho+1}u_1;$$

$$f_2(u_1, u_2) = |u_1 + u_2|^{2\rho}(u_1 + u_2) + |u_1|^{\rho+1}|u_2|^{\rho-1}u_2;$$

$$\Delta_q u = \sum_{i=1}^{\infty} \frac{\partial}{\partial x_i}\Big(\Big|\frac{\partial u}{\partial x_i}\Big|^q \frac{\partial u}{\partial x_i}\Big), \quad \text{where} \quad q \geq 2;$$

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$$(-\Delta)^{\alpha}u = \sum_{j=1}^{\infty} \lambda_j^{\alpha}(u, \varphi_j)\varphi_j,$$

where $0 < \lambda_1 < \lambda_2 \le \lambda_3 \le \ldots,\quad \varphi_1, \varphi_2, \varphi_3, \ldots$ are the sequence of eigenvalues and eigenfunctions of $-\Delta$ in $H_0^1(\Omega)$, respectively.

The norm in $L_q(\Omega)$ is denoted by $\|\cdot\|_q$ and in $\overset{\circ}{W}{}_q^1(\Omega) = \{u : u \in W_q^1(\Omega)\}$ we use the norm

$$\|u\|_{q,1}^q = \sum_{j=1}^{n} \|u_{x_j}\|_q^q.$$

For the reader's convenience, we recall some of the basic properties of the operators used here. The degenerate operator $\Delta_q u$ is bounded, monotone and hemicontinuous from $\overset{\circ}{W}{}_q^1(\Omega)$ to $W_{q'}^{-1}(\Omega)$, where $\frac{1}{q} + \frac{1}{q'} = 1$.

We investigate the nonexistence of the global solutions of problem (1.1)-(1.3).

2 Preliminaries and main results

First, we make some preparations. Assume $0 < \alpha \le 1$ and the ρ, q satisfy the condition

$$\rho \ge \frac{q-1}{2}; \tag{2.1}$$

$$0 < \rho < +\infty \quad \text{for} \quad n \le q; \tag{2.2}$$

$$0 < \rho < \frac{nq}{2(n-q)-1}, \quad \text{for} \quad n > q. \tag{2.3}$$

Lemma 2.1 *(Local existence) Let the condition* (2.1)-(2.3) *hold, then for any initial data* $u_{j0} \in \overset{\circ}{W}{}_q^1(\Omega)$, $u_{j1} \in L_2(\Omega)$, *if* T *is small enough, then there exists a solution* $(u_1, u_2)u$ *of* (1.1)-(1.3) *which satisfies*

$$u_j \in L_\infty(0, T; \overset{\circ}{W}{}_q^1(\Omega)),\ u_j' \in L_\infty(0, T; L_2(\Omega)) \cap L_2(0, T; D((-\Delta)^{\alpha/2})). \tag{2.4}$$

For the week solution the following energetic identity is valid

$$E(t) + \frac{1}{2}\sum_{j=1}^{2} \int_0^t \int_\Omega |u_j(t,x)|^2 dx = E(0), \tag{2.5}$$

where

$$E(t) = \frac{1}{2}\sum_{j=1}^{2} \int_\Omega |u_{jt}(t,x)|^2 dx + \frac{1}{q}\sum_{j=1}^{2} |\nabla u_j(t,x)|^q dx - G(u_1(t,x), u_2(t,x)), \tag{2.6}$$

$$G(u_1, u_2) = \frac{1}{2(p+1)} \int_\Omega |u_1(t,x) + u_2(t,x)|^{2(p+1)} dx + \frac{1}{p+1} \int_\Omega |u_1(t,x)u_2(t,x)|^{p+1} dx. \tag{2.7}$$

To prove the Lemma 2.1 we use the Galerkin method (see [8]).

In [1] is proved that in the case when conditions (2.2), (2.3) and

$$\rho < \frac{q-1}{2} \tag{2.8}$$

hold, then local solution defined by Lemma 2.1 is global. In this work we prove that for the global solvability condition (2.8) is principle.

Similar problems for the Klein-Gordon systems with the usual dissipations, i.e. when $\alpha = 0$ are enough investigated (see [2,3]).

For the semilinear equation with p-laplacian and with fractal dissipation the close results are obtained in [4,5].

When $q = 2$, and the damping term is given by $|u_t|^r u_t$, $r \geq 0$, many authors studied the existence and uniqueness of the global solution and the blow up of the solution (see [6,7]).

Our main results are expressed by the following theorem.

Theorem 2.1 *Let the conditions* (2.1)-(2.3) *hold, for any initial data* $(\varphi_j, \psi_j) \in \overset{\circ}{W}{}_q^1 (\Omega) \times L_2(\Omega)$, *if*

$$E(0) = \frac{1}{2}\sum_{j=1}^{2}\int_{\Omega}|\psi_j(x)|^2 + \frac{1}{q}\sum_{j=1}^{2}|\nabla\varphi_j(x)|^q dx - G(\varphi_1(x), \varphi_2(x)) < 0. \tag{2.9}$$

Then exists $T' > 0$ *such that*

$$\lim_{t \to T'-0}\sum_{j=1}^{2}\|u_j(t,\cdot)\|_{2(p+1)} = +\infty,$$

where $0 < T' \leq T_0$ *(*T_0 *can be seen in the proof).*

Proof of Theorem 2.1. Using (2.5) and (2.9), we have

$$E(t) \leq E(0) < 0. \tag{2.10}$$

Let

$$H(t) = q(-E(t)) + \left(\frac{q}{2}+1\right)\sum_{j=1}^{2}\int_{\Omega}|u_{jt}|^2 dx + \frac{1}{q}\sum_{j=1}^{2}\int_{\Omega}|\nabla u_j|^q dx + \frac{2(p+1)-q}{2}G(u_1,u_2). \tag{2.11}$$

Now, we introduce

$$F(t) = \frac{1}{2}\sum_{j=1}^{2}\int_{\Omega}|u_j(t,x)|^2 dx$$

for any solution u, then differentiate $F(t)$ with respect to t we have

$$F'(t) = \sum_{j=1}^{2}\int_{\Omega}u_j(t,x)u_{jt}(t,x)dx. \tag{2.12}$$

The following Statement is valid (see [9]).

Lemma 2.2 *There exists $C_1 > 0$, $C_2 > 0$ such that the inequality*

$$C_1 \sum_{j=1}^{2} \|\nu_j\|_{2(p+1)}^{2(p+1)} \le \int_{\Omega} G(\nu_1, \nu_2)dx \le C_2 \sum_{j=1}^{2} \|\nu_j\|_{2(p+1)}^{2(p+1)}$$

is valid for all $\nu_1, \nu_2 \in L_{2(p+1)}(\Omega)$.

From (2.5) follows that

$$\frac{1}{q} \sum_{j=1}^{2} \|\nabla u_j(t,\cdot)\|_q^q \le G(u_1(t,x), u_2(t,x)). \tag{2.13}$$

Considering Lemma 2.2 from this we get

$$\sum_{j=1}^{2} \|\nabla u_j(t,\cdot)\|_q^q \le qC_2 \sum_{j=1}^{2} \|u_j(t,\cdot)\|_{2(p+1)}^{2(p+1)}. \tag{2.14}$$

Using (1.1), (1.2), (2.5), (2.6) and (2.12) we obtain that

$$\begin{aligned} F''(t) &= \sum_{j=1}^{2} \int_{\Omega} |u_{jt}(t,x)|^2 dx - \sum_{j=1}^{2} \int_{\Omega} |\nabla u_j(t,x)|^q dx + 2(p+1) \int_{\Omega} G(u_1(t,x), u_2(t,x))dx \\ &+ \sum_{j=1}^{2} \int_{\Omega} (-\Delta)^{\alpha} u_{jt}(t,x) u_j(t,x)dx = q(-E(t)) + \Big(1 + \frac{q}{2}\Big) \sum_{j=1}^{2} \int_{\Omega} |u_{jt}(t,x)|^2 dx \\ &+ [2(p+1) - q] \int_{\Omega} G(u_1(t,x), u_2(t,x))dx + \sum_{j=1}^{2} \int_{\Omega} (-\Delta)^{\alpha} u_{jt}(t,x) u_j(t,x)dx. \end{aligned}$$

Due to (2.11) we have the inequality

$$\begin{aligned} F''(t) \ge H(t) &+ \frac{2(p+1) - q}{2} \int_{\Omega} G(u_1(t,x), u_2(t,x))dx \\ &+ \sum_{j=1}^{2} \int_{\Omega} (-\Delta)^{\alpha} u_{jt}(t,x) u_j(t,x)dx. \end{aligned} \tag{2.15}$$

Then considering Lemma 2.2 again we obtain

$$\begin{aligned} F''(t) \ge H(t) &+ \frac{2(p+1) - q}{2} C_1 \sum_{j=1}^{2} \|u_j(t,\cdot)\|_{2(p+1)}^{2(p+1)} \\ &+ \sum_{j=1}^{2} \int_{\Omega} (-\Delta)^{\alpha} u_{jt}(t,x) u_j(t,x)dx. \end{aligned} \tag{2.16}$$

By virtue of (2.1)-(2.3), we have

$$\Big\|(-\Delta)^{\frac{\alpha}{2}} u_j(t,\cdot)\Big\|_2 \le C_3 \Big\|\nabla u_j(t,\cdot)\Big\|_q^q. \tag{2.17}$$

It follows from (2.5) that

$$G(u_1, u_2) \ge -E(t). \tag{2.18}$$

Due to lemma 2.2 there exists $C_4 > 0$ that provides

$$C_4 \sum_{j=1}^{2} \|u_j\|_{2(p+1)}^{2(p+1)} \geq \sum_{j=1}^{2} \|\nabla u_j\|_q^q. \tag{2.19}$$

From (2.19) and (2.20) we get

$$C_5 \sum_{j=1}^{2} \|u_j\|_{2(p+1)}^{2(p+1)} \geq -E(t), \tag{2.20}$$

where $C_5 > 0$ is some constant. From the last follows

$$C_6 \sum_{j=1}^{2} \|u_j\|_{2(p+1)} \geq (-E(t))^{\frac{1}{2(p+1)}}, \tag{2.21}$$

$$G(u_1, u_2) \geq C_1 \sum_{j=1}^{2} \|\nabla u_j\|_q^q \geq C_7 \Big(\sum_{j=1}^{2} \|\nabla u_j\|_q \Big)^q, \tag{2.22}$$

where $C_6 > 0$, $C_7 > 0$ are some constants.

By virtue of (2.3) $1 < 2(p+1) < \frac{nq}{n-q}$. So $W_{q,0}^1(\Omega) \subset L_{2(p+1)}(\Omega)$, i.e.

$$\|\nu\|_{2(p+1)} \leq C_8 \|\nabla \nu\|_q \quad \text{(see [10])}. \tag{2.23}$$

Therefore from (2.21) and (2.22) we get the inequality

$$C_6 C_8 \sum_{j=1}^{2} \|\nabla u_j\|_q \geq (-E(t))^{\frac{1}{2(p+1)}}. \tag{2.24}$$

By virtue of (2.1)-(2.3) $\frac{2-q}{2} \leq 0$, therefore from (2.24) we get

$$C_9 \Big(\sum_{j=1}^{2} \|\nabla u\|_q \Big)^{\frac{2-q}{2}} \leq (-E(t))^{\frac{2-q}{4(p+1)}}, \tag{2.25}$$

where $C_9 = (C_6 C_8)^{\frac{2-q}{2}}$.

Denoting $\beta = \frac{2-q}{4(p+1)} + 1 = \frac{4p+6-q}{4(p+1)}$ we have $-(1-\beta) = \frac{2-q}{4(p+1)}$.

Further, using embedding theorems and taking into account conditions (2.1)-(2.3), (2.10) and (2.25), we obtain the following

$$\Big|\sum_{j=1}^{2}\int_{\Omega}(-\Delta)^{\alpha}u_{jt}(t,x)u_j(t,x)dx\Big| = \sum_{j=1}^{2}\Big\|(-\Delta)^{\frac{\alpha}{2}}u_{jt}\Big\|_2\cdot\Big\|(-\Delta)^{\frac{\alpha}{2}}u_j\Big\|_2$$
$$\le \sum_{j=1}^{2}\Big\|(-\Delta)^{\frac{\alpha}{2}}u_{jt}\Big\|_2\cdot\sum_{j=1}^{2}\Big\|(-\Delta)^{\frac{\alpha}{2}}u_j\Big\|_2$$
$$\le C_{10}\sum_{j=1}^{2}\Big\|(-\Delta)^{\frac{\alpha}{2}}u_{jt}\Big\|_2\cdot\sum_{j=1}^{2}\Big\|\nabla u_j\Big\|_q$$
$$= C_{10}\Big\|(-\Delta)^{\frac{\alpha}{2}}u_{jt}\Big\|_2\cdot\Big(\sum_{j=1}^{2}\Big\|\nabla u_j\Big\|_q\Big)^{\frac{q}{2}}\Big(\sum_{j=1}^{2}\Big\|\nabla u_j\Big\|_q\Big)^{1-\frac{q}{2}}$$
$$\le C_{10}\Big[\frac{1}{2\varepsilon}\sum_{j=1}^{2}\Big\|(-\Delta)^{\frac{\alpha}{2}}u_j\Big\|_2^2+2\varepsilon\sum_{j=1}^{2}\Big\|\nabla u_j\Big\|_q^q\Big]\cdot\Big(\sum_{j=1}^{2}\Big\|\nabla u_j\Big\|_q\Big)^{1-\frac{q}{2}}$$
$$\le \frac{C_9C_{10}}{2\varepsilon}(-E(t))^{-(1-\beta)}\sum_{j=1}^{2}\Big\|(-\Delta)^{\frac{\alpha}{2}}u_{jt}\Big\|_2^2$$
$$+2\varepsilon C_9C_{10}\Big(\sum_{j=1}^{2}\Big\|\nabla u_j\Big\|_q\Big)^q(-E(0))^{-(1-\beta)}. \tag{2.26}$$

Setting $\varepsilon=\frac{(2(p+1)-q)(-E(0))^{1-\beta}}{4C_9C_{10}}$ we get

$$F''(t)\ge H(t)-C_9C_{10}(-E(t))^{-(1-\beta)}\sum_{j=1}^{2}\Big\|(-\Delta)^{\frac{\alpha}{2}}u_{jt}\Big\|_2^2. \tag{2.27}$$

Introduce the functional

$$\Phi(t)=(-E(t))^{\beta}+\beta\frac{2\varepsilon}{C_9C_{10}}F'(t). \tag{2.28}$$

Using (2.27) and (2.28) we obtain

$$\Phi'(t)=\beta(-E(t))^{-(1-\beta)}(-E'(t))+\beta\frac{2\varepsilon}{C_9C_{10}}F''(t)\ge\beta(-E(t))^{-(1-\beta)}(-E'(t))$$
$$+\beta\frac{2\varepsilon}{C_9C_{10}}\Big[H(t)-\frac{C_9C_{10}}{2\varepsilon}(-E(t))^{-(1-\beta)}\sum_{j=1}^{2}\Big\|(-\Delta)^{\frac{\alpha}{2}}u_{jt}\Big\|\Big]=\frac{2\beta\varepsilon}{C_9C_{10}}H(t)>0. \tag{2.29}$$

Assume (2.9) holds, then

$$\Phi(0)=(-E(0))^{\beta}+\beta\frac{2\varepsilon}{C_9C_{10}}\sum_{j=1}^{2}\int_{\Omega}\varphi_j(x)\psi_j(x)dx>0. \tag{2.30}$$

By (2.29) and (2.30), we have

$$\Phi(t)>\Phi(0)>0.$$

Considering this from (2.28) it is easy to get

$$\Phi^{1/\beta}(t) \leq 4\Big\{(-E(t)) + \Big(\beta\frac{2\varepsilon}{C_9C_{10}}F'(t)\Big)^{1/\beta}\Big\}. \tag{2.31}$$

From other side

$$\begin{aligned}|F'(t)| &\leq \sum_{j=1}^{2}\Big\|u_{jt}\Big\|^{\frac{2(p+1)}{2p+1}}_{\frac{2(p+1)}{2p+1}}\|u_j\|_{2(p+1)}\\ &\leq \sum_{j=1}^{2}\|u_{jt}\|_2\|u_j\|_{2(p+1)} \leq \sum_{j=1}^{2}\Big\|u_{jt}\Big\|_2\sum_{j=1}^{2}\Big\|u_j\Big\|_{2(p+1)}.\end{aligned} \tag{2.32}$$

Taking into account (2.26) and (2.32), from (2.31) we obtain

$$\begin{aligned}\Phi^{1/\beta}(t) &\leq 4\Big\{(-E(t)) + \Big(\beta\frac{2\varepsilon}{C_9C_{10}}F'(t)\Big)^{1/\beta}\Big\}\\ &\leq 4\Big\{(-E(t)) + \Big(\beta\frac{2\varepsilon}{C_9C_{10}}\sum_{j=1}^{2}\Big\|u_{jt}\Big\|_2\sum_{j=1}^{2}\Big\|u_j\Big\|_{2(p+1)}\Big)^{1/\beta}\Big\}\\ &\leq 4\Big\{(-E(t)) + \frac{1}{2}\Big(\sum_{j=1}^{2}\Big\|u_{jt}\Big\|_2\Big)^2 + \Big(\beta\frac{2\sqrt{2}\varepsilon}{C_9C_{10}}\sum_{j=1}^{2}\Big\|u_j\Big\|_{2(p+1)}\Big)^{\frac{2}{2\beta-1}}\Big\}\\ &\leq 8\Big\{(-E(t)) + \frac{1}{2}\sum_{j=1}^{2}\int_\Omega|u_{jt}(t,x)|^2dx + \Big(\beta\frac{2\sqrt{2}\varepsilon}{C_9C_{10}}\sum_{j=1}^{2}\Big\|u_j\Big\|_{2(p+1)}\Big)^{\frac{2}{2\beta-1}}\Big\}\end{aligned}$$

By virtue of Lemma 2.2 and (2.11)

$$C_2\frac{2(p+1)-q}{2}\sum_{j=1}^{2}\|u_j\|^{2(p+1)}_{2(p+1)} \leq H(t).$$

Therefore

$$\Phi^{1/\beta}(t) \leq 8\Big\{(-E(t)) + \frac{1}{2}\sum_{j=1}^{2}\int_\Omega|u_{jt}(t,x)|^2dx + \Big(\beta\frac{2\sqrt{2}\varepsilon}{C_9C_{10}}\sum_{j=1}^{2}\Big\|u_j\Big\|_{2(p+1)}\Big)^{\frac{2}{2\beta-1}}\Big\}. \tag{2.33}$$

From (2.9) and (2.9) we get

$$C_{12}\sum_{j=1}^{2}\Big\|u_j\Big\|_{2(p+1)} \geq -E(0).$$

It follows from here that

$$(-C_{12}E(0))^{2(p+1)-\frac{2}{2\beta-1}}\sum_{j=1}^{2}\Big\|u_j\Big\|^{2(p+1)-\frac{2}{2\beta-1}}_{2(p+1)} \geq 1.$$

Consideration this in (2.33) gives

$$\begin{aligned}\Phi^{1/\beta}(t) \le 8\Big\{(-E(t)) + \frac{1}{2}\sum_{j=1}^{2}\int_{\Omega}|u_{jt}(t,x)|^2dx \\ + \Big(\beta\frac{2\varepsilon}{C_9C_{10}}\sum_{j=1}^{2}\|u_j\|_{2(p+1)}\Big)^{\frac{2}{2\beta-1}}(-C_{12}E(0))^{2(p+1)-\frac{2}{2\beta-1}}\sum_{j=1}^{2}\Big\|u_j\Big\|_{2(p+1)}^{2(p+1)-\frac{2}{2\beta-1}}\Big\} \\ \le C_{13}H(t). \qquad (2.34)\end{aligned}$$

Using (2.29) and (2.34) we get

$$\frac{d}{dt}\Phi^{1-\frac{1}{\beta}}(t) = \Big(1-\frac{1}{\beta}\Big)\Phi^{-\frac{1}{\beta}}(t)\Phi'(t) \le -C_{14},$$

where $C_{14} = \frac{2\varepsilon(1-\beta)}{C_9C_{10}C_{13}} > 0$. Therefore we have

$$\Phi(t) \ge \Big[\frac{\Phi^{\frac{1-\beta}{\beta}}(0)}{1 - C_{14}\Phi^{\frac{1-\beta}{\beta}}(0)\cdot t}\Big]^{\frac{\beta}{1-\beta}}.$$

Hence there exists a $T' > 0$ such that

$$T' \le T_0 = \frac{1}{C_{14}\Phi^{\frac{1-\beta}{\beta}}(0)}$$

and

$$\lim_{t\to T''-0}\Phi(t) = +\infty. \qquad (2.35)$$

By virtue of the energetic inequality

$$(-E(t)) + \frac{1}{2}\sum_{j=1}^{2}\int_{\Omega}|u_{jt}(t,x)|^2dx = \frac{1}{q}\sum_{j=1}^{2}|\nabla u_j(t,x)|^q dx - \int_{\Omega}G(u_1(t,x),u_2(t,x))dx.$$

From the last

$$\begin{aligned}(-E(t)) + \frac{1}{2}\sum_{j=1}^{2}\int_{\Omega}|u_{jt}(t,x)|^2dx \le \int_{\Omega}G(u_1(t,x),u_2(t,x))dx \\ \le C_2\sum_{j=1}^{2}\|u_j\|_{2(p+1)}^{2(p+1)}. \qquad (2.36)\end{aligned}$$

It follows from (2.35) and (2.36) that

$$\lim_{t\to T'-0}\sum_{j=1}^{2}\|u_j\|_{2(p+1)} = +\infty.$$

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