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Weakly semi-I-open sets in ideal bitopological spaces

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Abstract. *Semi-open sets play vital roles in topology. In this paper, our main motive is to define the concept of weakly semi-open set by using the notion of ideal in bitopological space. Here, we try to establish relation between* (σ_i, σ_j)*-semi-I-open sets and* (σ_i, σ_j)*-weakly semi-open sets. Moreover, it is shown that every* (σ_i, σ_j)*-semi-I-open set is* (σ_i, σ_j)*-weakly semi-open set but the converse is not true in general. We also study the relation of* (σ_i, σ_j)*-weakly semi-I-open set with* (σ_i, σ_j)*- semi-open set and* (σ_i, σ_j)*-preopen set.*

Keywords. (σ_i, σ_j)-wsI-open, (σ_i, σ_j)-semi-I-open, (σ_i, σ_j)-β-open, (σ_i, σ_j)-preopen.

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1 Introduction

In 1963, Kelly [10] introduced the definition of bitopological space (X, σ_1, σ_2), with respect to a non-empty X and two topologies σ_1 and σ_2. The definition of ideal in a topological space was defined by Kuratowski [12] and further it was studied by Vaidyanathasamy [16], Jankovic and Hamlett [8] and many others. If I is an ideal on the non-empty set X, then $(X, \sigma_1, \sigma_2, I)$ is called an ideal bitopological space. Using the concept of ideal, various open sets such as (σ_i, σ_j)-semi-I-open [4], (σ_i, σ_j)-bI-open [13], (σ_i, σ_j)-α-I-open [4], etc. have been introduced and studied in bitopological space. One may refer to Tripathy and Sarma [14], Tripathy and Debnath [15] for some of the latest works in the direction of b-open set and its generalizations. Hatir and Jafari [3] defined weakly semi-I-open sets in a topological space. Recently, Acharjee et al. [2] studied various results of compactness via bI-open sets in an ideal bitopological space. Moreover, bitopological space and related generalized forms have started to find applications in many areas of science and social science e.g. economics by Acharjee and Tripathy [3], theoretical computer science by Jakl et al. [7], etc. Thus, various generalized forms in bitopology may be useful in the future.

In this paper, we introduce the notion of weakly semi-I-open sets in ideal bitopological space and investigate various properties of it.

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We write the notions of interior and closure of $P \subseteq X$ in $(X, \sigma_1, \sigma_2, I)$ with respect to the topology σ_i by σ_i-$int(P)$ and σ_i-$cl(P)$ respectively, for $i \in \{1, 2\}$. Also, the pair (σ_i, σ_j) denotes the pairwise topologies σ_i and σ_j, in which $i, j \in \{1, 2\}$ but $i \neq j$.

Now, we recall some existing definitions and results which will be used in this paper.

According to Kuratowski [12], a collection $I \subseteq P(X)$, where $P(X)$ is power set of X in (X, σ) is an ideal on X if it satisfies following two conditions:
(i) $Q \in I$, $R \subseteq Q$ implies $R \in I$,
(ii) $Q \in I$, $R \in I$ implies $Q \cup R \in I$.

In (X, σ, I) , the operator $(.)^* : P(X) \longrightarrow P(X)$ is said to be a local function [10] of $Q \subseteq X$ due to topology σ and ideal I. We write $Q^*(\sigma, I) = \{y \in X : R \cap Q \notin I$, for all $R \in \sigma(y)\}$, in which $Q \subseteq X$ and $\sigma(y) = \{M \in \sigma : y \in M\}$. Instead of $Q^*(\sigma, I)$, we simply write Q^*. For $\sigma^*(I)$ which is finer than the topology σ, a Kuratowski closure operator is defined by $cl^*(Q) = Q \cup Q^*$. Also, σ_i-$int^*(Q)$ is the interior of Q in $\sigma_i^*(I)$ and σ_i-$int(Q_j^*)$ is the interior of Q_j^* due to σ_i , in which $Q_j^* = \{y \in X : M \cap Q \notin I$, for all $M \in \sigma_j(y)\}$.

Definition 1.1 *A subset $P \subseteq X$ in (X, σ_1, σ_2) is said to be*
(i) (σ_i, σ_j)-preopen [9] if $P \subseteq \sigma_i$-$int(\sigma_j$-$cl(P))$,
(ii) (σ_i, σ_j)-β-open [10] if $P \subseteq \sigma_j$-$cl(\sigma_i$-$int(\sigma_j$-$cl(P)))$.

Definition 1.2 *[4] A subset $P \subseteq X$ in $(X, \sigma_1, \sigma_2, I)$ is said to be*
(i) (σ_i, σ_j)-semi-I-open if $P \subseteq \sigma_j$-$cl^*(\sigma_i$-$int(P))$,
(ii) (σ_i, σ_j)-α-I-open if $P \subseteq \sigma_i$-$int(\sigma_j$-$cl^*(\sigma_i$-$int(P)))$.

Lemma 1.1 *[8] If R and S are any two subsets of (X, σ, I), then*
(i) if $R \subseteq S$, then $R^* \subseteq S^*$,
(ii) if $S \in \sigma$, then $S \cap R^* \subset (S \cap R)^*$,
(iii) R^* is closed in (X, σ).

2 (σ_i, σ_j)-weakly semi-I-open sets

In this section, we introduce (σ_i, σ_j)-weakly semi-I-open set and study some of its properties.

Definition 2.1 *A subset P in $(X, \sigma_1, \sigma_2, I)$ is called (σ_i, σ_j)-weakly semi-I open (in short, (σ_i, σ_j)-wsI-open) if $P \subseteq \sigma_j$-$cl^*(\sigma_i$-$int(\sigma_j$-$cl(P)))$.*

We denote the collection of (σ_i, σ_j)-wsI-open sets in X by (σ_i, σ_j)-$WSIO(X)$.

Remark 2.1 If I and K be two ideals in (X, σ_1, σ_2) such that $I \subseteq K$, then (σ_i, σ_j)-$WSKO(X) \subseteq (\sigma_i, \sigma_j)$-$WSIO(X)$.

Remark 2.2 Every (σ_i, σ_j)-semi-I-open set (in short (σ_i, σ_j)-sI-open set) in $(X, \sigma_1, \sigma_2, I)$ is (σ_i, σ_j)-wsI-open.

We provide the following example to show that converse of Remark 2.2 may not be true in general.

Example 1 Let, $X = \{u, v, w\}$, $\sigma_1 = \{\emptyset, \{v\}, X\}$, $\sigma_2 = \{\emptyset, \{v\}, \{v, w\}, X\}$ and $I = \{\emptyset, \{v\}\}$. Then, $\{v, w\}$ is (σ_1, σ_2)-wsI-open but not (σ_1, σ_2)-sI-open.

Now, we state some theorems related to (σ_i, σ_j)-wsI-open set.

Theorem 2.1 *Let, $(X, \sigma_1, \sigma_2, I)$ be an ideal bitopological space such that $P, Q \subseteq X$. If P is (σ_i, σ_j)-wsI-open and $Q \in \sigma_1 \cap \sigma_2$, then $P \cap Q$ is also (σ_i, σ_j)-wsI-open.*

Proof. Let, P be (σ_i, σ_j)-wsI-open. Then, $P \subseteq \sigma_j$-$cl^*(\sigma_i$-$int(\sigma_j$-$cl(P)))$.
Now, $P \cap Q \subseteq \sigma_j$-$cl^*(\sigma_i$-$int(\sigma_j$-$cl(P))) \cap Q$
= $((\sigma_i$-$int(\sigma_j$-$cl(P))) \cup (\sigma_i$-$int(\sigma_j$-$cl(P)))^*_j) \cap Q$
$\subseteq ((\sigma_i$-$int(\sigma_j$-$cl(P))) \cap Q) \cup ((\sigma_i$-$int(\sigma_j$-$cl(P)))^*_j \cap Q)$
$\subseteq (\sigma_i$-$int(\sigma_j$-$cl(P \cap Q))) \cup ((\sigma_i$-$int(\sigma_j$-$cl(P))) \cap Q)^*_j$
$\subseteq (\sigma_i$-$int(\sigma_j$-$cl(P \cap Q))) \cup (\sigma_i$-$int(\sigma_j$-$cl(P \cap Q)))^*_j$
= σ_j-$cl^*(\sigma_i$-$int(\sigma_j$-$cl(P \cap Q)))$.
Hence, $P \cap Q$ is (σ_i, σ_j)-wsI-open.

Theorem 2.2 *Union of arbitrary collection of (σ_i, σ_j)-wsI-open sets in $(X, \sigma_1, \sigma_2, I)$ is (σ_i, σ_j)-wsI-open.*

Proof. Let, $P_\delta \in (\sigma_i, \sigma_j)$-$WSIO(X)$. Then, we have $P_\delta \subseteq \sigma_j$-$cl^*(\sigma_i$-$int(\sigma_j$-$cl(P_\delta)))$, where $\delta \in \wedge$ and $\wedge$ is an index set.
Now, $\bigcup_{\delta\in\wedge} P_\delta \subseteq \bigcup_{\delta\in\wedge}\{\sigma_j$-$cl^*(\sigma_i$-$int(\sigma_j$-$cl(P_\delta)))\}$.
$\subseteq \bigcup_{\delta\in\wedge}\{(\sigma_i$-$int(\sigma_j$-$cl(P_\delta))) \cup (\sigma_i$-$int(\sigma_j$-$cl(P_\delta)))^*_j\}$
$\subseteq (\sigma_i$-$int(\sigma_j$-$cl(\bigcup_{\delta\in\wedge} P_\delta))) \cup (\bigcup_{\delta\in\wedge}(\sigma_i$-$int(\sigma_j$-$cl(P_\delta)))^*_j)$
$\subseteq (\sigma_i$-$int(\sigma_j$-$cl(\bigcup_{\delta\in\wedge} P_\delta))) \cup (\sigma_i$-$int(\sigma_j$-$cl(\bigcup_{\delta\in\wedge} P_\delta)))^*_j$
$\subseteq \sigma_j$-$cl^*(\sigma_i$-$int(\sigma_j$-$cl(\bigcup_{\delta\in\wedge} P_\delta)))$
Consequently, $\bigcup_{\delta\in\wedge} P_\delta$ is (σ_i, σ_j)-wsI-open.

The following lemma states the relation between relative topology and local function with respect to the concept of pairwise in bitopology.

Lemma 2.1 *If R and S be two subsets of $(X, \sigma_1, \sigma_2, I)$ such that $S \subseteq R$. Then $S^*_i(\sigma_i|_R, I|_R) = S^*_i(\sigma_i, I) \cap R$, where $i \in \{1, 2\}$.*

Here, $\sigma_i|_R$ denotes the relative topology on R and $I|_R = \{R \cap K : K \in I\}$ is an ideal on R.

Theorem 2.3 *If $(X, \sigma_1, \sigma_2, I)$ be an ideal bitopological space such that $P \in (\sigma_i, \sigma_j)$-$WSIO(X)$ and $Q \in \sigma_1 \cap \sigma_2$, then $P \cap Q \in WSIO(Q, \sigma_1|_Q, \sigma_2|_Q, I|_Q)$.*

Proof. Since $Q \in \sigma_1 \cap \sigma_2$, therefore σ_i-$int_Q(S) = \sigma_i$-$int(S)$, where $S \subseteq Q$ and $i \in \{1, 2\}$. Applying this result and Lemma 2.1, we see that
$P \cap Q \subseteq \sigma_j$-$cl^*(\sigma_i$-$int(\sigma_j$-$cl(P))) \cap Q$
$= \{(\sigma_i$-$int(\sigma_j$-$cl(P))) \cup (\sigma_i$-$int(\sigma_j$-$cl(P)))^*_j) \cap Q\}$
$\subseteq \{(\sigma_i$-$int(\sigma_j$-$cl(P)) \cap Q) \cup ((\sigma_i$-$int(\sigma_j$-$cl(P)))^*_j \cap Q) \cap Q\}$
$\subseteq \{(\sigma_i$-$int(\sigma_j$-$cl(P)) \cap Q) \cup ((\sigma_i$-$int(\sigma_j$-$cl(P))) \cap Q)^*_j) \cap Q\}$
$\subseteq \{(\sigma_i$-$int_Q(\sigma_j$-$cl_Q(P \cap Q)) \cap Q) \cup (\sigma_i$-$int_Q(\sigma_j$-$cl_Q(P \cap Q)^*_j)) \cap Q\}$
$\subseteq \{(\sigma_i$-$int_Q(\sigma_j$-$cl_Q(P \cap Q))) \cup (\sigma_i$-$int_Q(\sigma_j$-$cl_Q(P \cap Q))^*_j)_{(\sigma_1|Q, \sigma_2|Q, I|Q)}\}$
$= \sigma_j$-$cl^*_Q(\sigma_i$-$int_Q(\sigma_j$-$cl_Q(P \cap Q)))$.
Hence, $P \cap Q \in WSIO(Q, \sigma_1|_Q, \sigma_2|_Q, I|_Q)$.

We state the following result, whose proof is straightforward.

Theorem 2.4 *In $(X, \sigma_1, \sigma_2, I)$, a subset $P \subseteq X$ is (σ_i, σ_j)-wsI-open if and only if for all $y \in X$, there exists $Q \in (\sigma_i, \sigma_j)$-$WSIO(X)$ such that $y \in Q \subseteq P$.*

It is easy to find interconnections between various generalized open sets in bitopology. Here, we have the following results

Theorem 2.5 *In $(X, \sigma_1, \sigma_2, I)$, the following results hold:*
(i) every (σ_i, σ_j)-wsI-open set is (σ_i, σ_j)-β-open,
(ii) every (σ_i, σ_j)-preopen set is (σ_i, σ_j)-wsI-open.

Proof. (i) Suppose, P be (σ_i, σ_j)-wsI-open. Then,
$P \subseteq \sigma_j\text{-}cl^*(\sigma_i\text{-}int(\sigma_j\text{-}cl(P)))$
$= (\sigma_i\text{-}int(\sigma_j\text{-}cl(P))) \cup (\sigma_i\text{-}int(\sigma_j\text{-}cl(P)))^*_j$
$\subseteq (\sigma_i\text{-}int(\sigma_j\text{-}cl(P))) \cup \sigma_j\text{-}cl(\sigma_i\text{-}int(\sigma_j\text{-}cl(P)))$
$= \sigma_j\text{-}cl(\sigma_i\text{-}int(\sigma_j\text{-}cl(P)))$.
Hence, P is (σ_i, σ_j)-β-open.
(ii) Let, P be (σ_i, σ_j)-preopen. Then, we have $P \subseteq \sigma_i\text{-}int(\sigma_j\text{-}cl(P)) \subseteq \sigma_j\text{-}cl^*(\sigma_i\text{-}int(\sigma_j\text{-}cl(P)))$.
Hence, P is (σ_i, σ_j)-wsI-open.

Thus, it is easy to find that (σ_i, σ_j)-wsI-open set is acting as factor of interconnection between (σ_i, σ_j)-preopen set and (σ_i, σ_j)-β-open.
Hence, (σ_i, σ_j)-preopen set $\Rightarrow$ (σ_i, σ_j)-wsI-open set $\Rightarrow$ (σ_i, σ_j)-β-open set.

Theorem 2.6 *If P is (σ_i, σ_j)-wsI-open in $(X, \sigma_1, \sigma_2, I)$ and $P \subseteq Q \subseteq \sigma_j\text{-}cl^*(P)$, then Q is also (σ_i, σ_j)-wsI-open.*

Proof. Let, P be (σ_i, σ_j)-wsI-open. So $P \subseteq \sigma_j\text{-}cl^*(\sigma_i\text{-}int(\sigma_j\text{-}cl(P)))$.
Since, $Q \subseteq \sigma_j\text{-}cl^*(P) \subseteq \sigma_j\text{-}cl^*(\sigma_j\text{-}cl^*(\sigma_i\text{-}int(\sigma_j\text{-}cl(P))))$
$= \sigma_j\text{-}cl^*(\sigma_i\text{-}int(\sigma_j\text{-}cl(P)))$
$\subseteq \sigma_j\text{-}cl^*(\sigma_i\text{-}int(\sigma_j\text{-}cl(Q)))$.
Hence, Q is (σ_i, σ_j)-wsI-open.

Theorem 2.7 *If $P \subseteq Y \subseteq X$ in $(X, \sigma_1, \sigma_2, I)$, where $P \in (\sigma_i, \sigma_j)\text{-}WSIO(Y)$ and $Y \in \sigma_1 \cap \sigma_2$, then $P \in (\sigma_i, \sigma_j)\text{-}WSIO(X)$.*

Proof. Let, $P \in (\sigma_i, \sigma_j)\text{-}WSIO(Y)$. Then,
$P \subseteq \sigma_j\text{-}cl^*_Y(\sigma_i\text{-}int_Y(\sigma_j\text{-}cl_Y(P)))$
$= \sigma_j\text{-}cl^*(\sigma_i\text{-}int_Y(\sigma_j\text{-}cl_Y(P))) \cap Y$
$\subseteq \sigma_j\text{-}cl^*(\sigma_i\text{-}int(\sigma_j\text{-}cl_Y(P)))$
$= \sigma_j\text{-}cl^*(\sigma_i\text{-}int(\sigma_j\text{-}cl(P) \cap Y))$
$\subseteq \sigma_j\text{-}cl^*(\sigma_i\text{-}int(\sigma_j\text{-}cl(P)))$
Hence, $P \in (\sigma_i, \sigma_j)\text{-}WSIO(X)$.

Lemma 2.2 *For a subset P in (X, τ_1, τ_2, I), $\sigma_j\text{-}cl^*(\sigma_i\text{-}int(\sigma_j\text{-}cl^*(\sigma_i\text{-}int(P)))) = \sigma_j\text{-}cl^*(\sigma_i\text{-}int(P))$.*

Following theorem shows the relation between (σ_i, σ_j)-α-I-open set and (σ_i, σ_j)-wsI-open set under intersection.

Theorem 2.8 *If P is (σ_i, σ_j)-α-I-open and Q is (σ_i, σ_j)-wsI-open in $(X, \sigma_1, \sigma_2, I)$, then $P \cap Q$ is also (σ_i, σ_j)-wsI-open.*

Proof. Let, P is (σ_i, σ_j)-α-I-open and Q is (σ_i, σ_j)-wsI-open in X. Therefore, we have $P \subseteq \sigma_i\text{-}int(\sigma_j\text{-}cl^*(\sigma_i\text{-}int(P)))$ and $Q \subseteq \sigma_j\text{-}cl^*(\sigma_i\text{-}int(\sigma_j\text{-}cl(Q)))$.
Now, $P \cap Q \subseteq \sigma_i\text{-}int(\sigma_j\text{-}cl^*(\sigma_i\text{-}int(P))) \cap \sigma_j\text{-}cl^*(\sigma_i\text{-}int(\sigma_j\text{-}cl(Q)))$
$\subseteq \sigma_j\text{-}cl^*(\sigma_i\text{-}int(\sigma_j\text{-}cl^*(\sigma_i\text{-}int(P))) \cap \sigma_i\text{-}int(\sigma_j\text{-}cl(Q)))$
$= \sigma_j\text{-}cl^*(\sigma_i\text{-}int(\sigma_j\text{-}cl^*(\sigma_i\text{-}int(P)) \cap \sigma_i\text{-}int(\sigma_j\text{-}cl(Q)))$
$\subseteq \sigma_j\text{-}cl^*(\sigma_i\text{-}int(\sigma_j\text{-}cl^*(\sigma_i\text{-}int(P) \cap \sigma_i\text{-}int(\sigma_j\text{-}cl(Q))))$
$= \sigma_j\text{-}cl^*(\sigma_i\text{-}int(\sigma_j\text{-}cl^*(\sigma_i\text{-}int(\sigma_i\text{-}int(P) \cap \sigma_j\text{-}cl(Q)))))$
$\subseteq \sigma_j\text{-}cl^*(\sigma_i\text{-}int(\sigma_j\text{-}cl^*(\sigma_i\text{-}int(\sigma_j\text{-}cl(\sigma_i\text{-}int(P) \cap Q)))))$
$\subseteq \sigma_j\text{-}cl^*(\sigma_i\text{-}int(\sigma_j\text{-}cl^*(\sigma_i\text{-}int(\sigma_j\text{-}cl(P \cap Q)))))$
$= \sigma_j\text{-}cl^*(\sigma_i\text{-}int(\sigma_j\text{-}cl(P \cap Q)))$.
Hence, $P \cap Q$ is (σ_i, σ_j)-wsI-open.

Definition 2.2 *In $(X, \sigma_1, \sigma_2, I)$, a subset $P \subseteq X$ is called (σ_i, σ_j)-I-locally closed if $P = Q \cap P_j^*$, where Q is σ_i-open in X.*

Following theorem states the relation between (σ_i, σ_j)-I-locally closedness, (σ_i, σ_j)-wsI-openness and (σ_i, σ_j)-$semi$-I-openness.

Theorem 2.9 *In $(X, \sigma_1, \sigma_2, I)$, a subset $P \subseteq X$ is both (σ_i, σ_j)-I-locally closed and (σ_i, σ_j)-wsI-open, then P is (σ_i, σ_j)-semi-I-open.*

Proof. Suppose, Q is σ_i-open. Since, P is (σ_i, σ_j)-wsI-open, so we have $P \subseteq \sigma_j\text{-}cl^*(\sigma_i\text{-}int(\sigma_j\text{-}cl(P)))$. Again, P is (σ_i, σ_j)-I-locally closed, thus

$P = Q \cap P_j^* \subseteq Q \cap (\sigma_j\text{-}cl^*(\sigma_i\text{-}int(\sigma_j\text{-}cl(P))))_j^*$
$\subseteq Q \cap \sigma_j\text{-}cl^*(\sigma_j\text{-}cl^*(\sigma_i\text{-}int(\sigma_j\text{-}cl(Q \cap P_j^*))))$
$= Q \cap \sigma_j\text{-}cl^*(\sigma_i\text{-}int(\sigma_j\text{-}cl(Q \cap P_j^*)))$
$\subseteq \sigma_j\text{-}cl^*(Q \cap \sigma_i\text{-}int(\sigma_j\text{-}cl(Q \cap P_j^*)))$
$= \sigma_j\text{-}cl^*(\sigma_i\text{-}int(Q \cap \sigma_j\text{-}cl(Q \cap P_j^*)))$
$\subseteq \sigma_j\text{-}cl^*(\sigma_i\text{-}int(Q \cap \sigma_j\text{-}cl(Q) \cap \sigma_j\text{-}cl(P_j^*)))$
$= \sigma_j\text{-}cl^*(\sigma_i\text{-}int(Q \cap P_j^*))$
$= \sigma_j\text{-}cl^*(\sigma_i\text{-}int(P))$.

Hence, P is (σ_i, σ_j)-semi-I-open.

Definition 2.3 *A subset $P \subseteq X$ of $(X, \sigma_1, \sigma_2, I)$ is called (σ_i, σ_j)-wsI-closed if $X \setminus P$ is (σ_i, σ_j)-wsI-open.*

Theorem 2.10 *A subset $P \subseteq X$ of $(X, \sigma_1, \sigma_2, I)$ is (σ_i, σ_j)-wsI-closed if and only if $\sigma_j\text{-}int^*(\sigma_i\text{-}cl(\sigma_j\text{-}int(P))) \subseteq P$.*

Proof. Let, P is (σ_i, σ_j)-wsI-closed. So, $X \setminus P$ is (σ_i, σ_j)-wsI-open and therefore, $X \setminus P \subseteq \sigma_j\text{-}cl^*(\sigma_i\text{-}int(\sigma_j\text{-}cl(X \setminus P))) = \sigma_j\text{-}cl^*(\sigma_i\text{-}int(X \setminus \sigma_j\text{-}int(P))) = \sigma_j\text{-}cl^*(X \setminus \sigma_i\text{-}cl(\sigma_j\text{-}int(P))) \subseteq \sigma_j\text{-}cl(X \setminus \sigma_i\text{-}cl(\sigma_j\text{-}int(P))) = X \setminus \sigma_j\text{-}int(\sigma_i\text{-}cl(\sigma_j\text{-}int(P))) \subseteq X \setminus \sigma_j\text{-}int^*(\sigma_i\text{-}cl(\sigma_j\text{-}int(P)))$. Hence, $\sigma_j\text{-}int^*(\sigma_i\text{-}cl(\sigma_j\text{-}int(P))) \subseteq P$.

Conversely, let $\sigma_j\text{-}int^*(\sigma_i\text{-}cl(\sigma_j\text{-}int(P))) \subseteq P$. Now, $X \setminus P \subseteq X \setminus \sigma_j\text{-}int^*(\sigma_i\text{-}cl(\sigma_j\text{-}int(P)))= \sigma_j\text{-}cl^*(\sigma_i\text{-}int(\sigma_j\text{-}cl(X \setminus P)))$. Therefore, $X \setminus P$ is (σ_i, σ_j)-wsI-open and hence, P is (σ_i, σ_j)-wsI-closed.

Theorem 2.11 *In $(X, \sigma_1, \sigma_2, I)$, if a subset $P \subseteq X$ is (σ_i, σ_j)-wsI-closed, then $\sigma_j\text{-}int(\sigma_i\text{-}cl^*(\sigma_j\text{-}int(P)) \subseteq P$.*

Proof. Since, $\sigma_i^*(I)$ is finer than σ_i where $i \in \{1, 2\}$, then $\sigma_j\text{-}int(\sigma_i\text{-}cl^*(\sigma_j\text{-}int(P)) \subseteq \sigma_j\text{-}int^*(\sigma_i\text{-}cl^*(\sigma_j\text{-}int(P))) \subseteq \sigma_j\text{-}int^*(\sigma_i\text{-}cl(\sigma_j\text{-}int(P))) \subseteq P$.

Hence, $\sigma_j\text{-}int(\sigma_i\text{-}cl^*(\sigma_j\text{-}int(P)) \subseteq P$.

Definition 2.4 *In $(X, \sigma_1, \sigma_2, I)$, a subset $P \subseteq X$ is called (σ_i, σ_j)-weakly semi-I-neighbourhood of a point y of X if there exists $Q \in (\sigma_i, \sigma_j)\text{-}WSIO(X)$ such that $y \in Q \subseteq P$.*

Theorem 2.12 *In $(X, \sigma_1, \sigma_2, I)$, a subset $P \subseteq X$ is (σ_i, σ_j)-wsI-open if and only if P is a (σ_i, σ_j)-weakly semi-I-neighbourhood of each of its points.*

Definition 2.5 *Let, $(X, \sigma_1, \sigma_2, I)$ be a space such that $P \subseteq X$. Then,*

(i) (σ_i, σ_j)-wsI-interior of P = $\bigcup\{Q : Q \subseteq P$ and $Q \in (\sigma_i, \sigma_j)\text{-}WSIO(X)\}$ and we denote it by $(\sigma_i, \sigma_j)\text{-}wsI\text{-}int(P)$.

(ii) (σ_i, σ_j)-wsI-closure of P = $\bigcap\{Q : P \subseteq Q$ and $Q \in (\sigma_i, \sigma_j)\text{-}WSIC(X)\}$ and we denote it by $(\sigma_i, \sigma_j)\text{-}wsI\text{-}cl(P)$.

Now, we state the following two results without proof.

Theorem 2.13 *For a subset $P \subseteq X$ in $(X, \sigma_1, \sigma_2, I)$, the following results hold:*
(i) (σ_i, σ_j)-wsI-$int(P)$ is (σ_i, σ_j)-wsI-open,
(ii) (σ_i, σ_j)-wsI-$cl(P)$ is (σ_i, σ_j)-wsI-closed,
(iii) P is (σ_i, σ_j)-wsI-open if and only if $P = (\sigma_i, \sigma_j)$-wsI-$int(P)$,
(iv) P is (σ_i, σ_j)-wsI-closed if and only if $P = (\sigma_i, \sigma_j)$-wsI-$cl(P)$.

Theorem 2.14 *For a subset $P \subseteq X$ in $(X, \sigma_1, \sigma_2, I)$, a point $y \in (\sigma_i, \sigma_j)$-$wsI$-$cl(P)$ if and only if $P \cap Q \neq \emptyset$, for all $Q \in (\sigma_i, \sigma_j)$-$WSIO(X)$ containing y.*

Theorem 2.15 *For a subset $P \subseteq X$ in $(X, \sigma_1, \sigma_2, I)$, the following results hold:*
(a) (σ_i, σ_j)-wsI-$cl(X \setminus P) = X \setminus (\sigma_i, \sigma_j)$-$wsI$-$int(P)$.
(b) (σ_i, σ_j)-wsI-$int(X \setminus P) = X \setminus (\sigma_i, \sigma_j)$-$wsI$-$cl(P)$.

Proof. (i) Let, $y \notin (\sigma_i, \sigma_j)$-$wsI$-$cl(X \setminus P)$. So, there exists a $Q \in (\sigma_i, \sigma_j)$-$WSIO(X)$ such that $y \in Q$ and $Q \cap (X \setminus P) = \emptyset$. Since $y \in Q$, therefore we have $y \notin X \setminus P$ and so, $y \in P$. Thus, we get $y \in Q \subseteq P$ and so $y \in (\sigma_i, \sigma_j)$-$wsI$-$int(P)$; which implies that $y \notin X \setminus (\sigma_i, \sigma_j)$-$wsI$-$int(P)$. Hence, $X \setminus (\sigma_i, \sigma_j)$-$wsI$-$int(P) \subseteq (\sigma_i, \sigma_j)$-$wsI$-$cl(X \setminus P)$.

Conversely, suppose that $y \notin X \setminus (\sigma_i, \sigma_j)$-$wsI$-$int(P)$. Then, we have $y \in (\sigma_i, \sigma_j)$-$wsI$-$int(P)$ and so, there exists a $Q \in (\sigma_i, \sigma_j)$-$WSIO(X)$ such that $y \in Q$ and $y \in Q \subseteq P$. Thus, we get $Q \cap (X \setminus P) = \emptyset$ and $y \notin (\sigma_i, \sigma_j)$-$wsI$-$cl(X \setminus P)$. Then, (σ_i, σ_j)-wsI-$cl(X \setminus P) \subseteq X \setminus (\sigma_i, \sigma_j)$-$wsI$-$int(P)$. Hence, (σ_i, σ_j)-wsI-$cl(X \setminus P) = X \setminus (\sigma_i, \sigma_j)$-$wsI$-$int(P)$.

(ii) It is similar to the proof of (i).

Theorem 2.16 *For a subset $P \subseteq X$ in $(X, \sigma_1, \sigma_2, I)$, the following results hold:*
(i) (σ_i, σ_j)-wsI-$int(P) = P \cap \sigma_j$-$cl^*(\sigma_i$-$int(\sigma_j$-$cl(P)))$,
(ii) (σ_i, σ_j)-wsI-$cl(P) = P \cup \sigma_j$-$int^*(\sigma_i$-$cl(\sigma_j$-$int(P)))$.

Proof. (a) Since $P \subseteq X$, therefore, we have $P \cap \sigma_j$-$cl^*(\sigma_i$-$int(\sigma_j$-$cl(P))) \subseteq \sigma_j$-$cl^*(\sigma_i$-$int(\sigma_j$-$cl(P))) = \sigma_j$-$cl^*(\sigma_i$-$int(\sigma_j$-$cl(P))) \cap \sigma_i$-$int(\sigma_j$-$cl(P))) \subseteq \sigma_j$-$cl^*(\sigma_i$-$int(\sigma_j$-$cl(P \cap \sigma_i$-$int(\sigma_j$-$cl(P))))) \subseteq \sigma_j$-$cl^*(\sigma_i$-$int(\sigma_j$-$cl(P \cup \sigma_j$-$cl^*(\sigma_i$-$int(\sigma_j$-$cl(P))))))$. Consequently, $P \cap \sigma_j$-$cl^*(\sigma_i$-$int(\sigma_j$-$cl(P)))$ is (σ_i, σ_j)-wsI-open set contained in P. Again, since (σ_i, σ_j)-wsI-$int(P)$ is the largest (σ_i, σ_j)-wsI-open set contained in P, therefore we have $P \cap \sigma_j$-$cl^*(\sigma_i$-$int(\sigma_j$-$cl(P))) \subseteq (\sigma_i, \sigma_j)$-$wsI$-$int(P)$.

Conversely, since (σ_i, σ_j)-wsI-$int(P)$ is (σ_i, σ_j)-wsI-open, therefore (σ_i, σ_j)-wsI-$int(P) \subseteq \sigma_j$-$cl^*(\sigma_i$-$int(\sigma_j$-$cl((\sigma_i, \sigma_j)$-$wsI$-$int(P)))) \subseteq \sigma_j$-$cl^*(\sigma_i$-$int(\sigma_j$-$cl(P)))$. So, we have (σ_i, σ_j)-wsI-$int(P) \subseteq P \cap \sigma_j$-$cl^*(\sigma_i$-$int(\sigma_j$-$cl(P)))$.

Hence, (σ_i, σ_j)-wsI-$int(P) = P \cap \sigma_j$-$cl^*(\sigma_i$-$int(\sigma_j$-$cl(P)))$.

(ii) Since, (σ_i, σ_j)-wsI-$cl(P)$ is (σ_i, σ_j)-wsI-closed set containing P, therefore we have σ_j-$int^*(\sigma_i$-$cl(\sigma_j$-$int(P))) \subseteq \sigma_j$-$int^*(\sigma_i$-$cl(\sigma_j$-$int((\sigma_i, \sigma_j)$-$wsI$-$cl(P)))) \subseteq (\sigma_i, \sigma_j)$-$wsI$-$cl(P)$. Thus, $P \cup \sigma_j$-$int^*(\sigma_i$-$cl(\sigma_j$-$int(P))) \subseteq P \cup (\sigma_i, \sigma_j)$-$wsI$-$cl(P) = (\sigma_i, \sigma_j)$-$wsI$-$cl(P)$. Consequently, $P \cup \sigma_j$-$int^*(\sigma_i$-$cl(\sigma_j$-$int(P))) \subseteq (\sigma_i, \sigma_j)$-$wsI$-$cl(P)$.

Conversely, we have σ_j-$int^*(\sigma_i$-$cl(\sigma_j$-$int(P \cup \sigma_j$-$int^*(\sigma_i$-$cl(\sigma_j$-$int(P)))))) \subseteq \sigma_j$-$int^*(\sigma_i$-$cl(\sigma_j$-$int(P \cup \sigma_i$-$cl(\sigma_j$-$int(P)))))) \subseteq \sigma_j$-$int^*(\sigma_i$-$cl(\sigma_j$-$int(P) \cup \sigma_i$-$cl(\sigma_j$-$int(P)))) = \sigma_j$-$int^*(\sigma_i$-$cl(\sigma_i$-$cl(\sigma_j$-$int(P)))) = \sigma_j$-$int^*(\sigma_i$-$cl(\sigma_j$-$int(P))) \subseteq P \cup \sigma_j$-$int^*(\sigma_i$-$cl(\sigma_j$-$int(P)))$. Therefore, $P \cup \sigma_j$-$int^*(\sigma_i$-$cl(\sigma_j$-$int(P)))$ is an (σ_i, σ_j)-wsI-closed set containing P. Since, (σ_i, σ_j)-wsI-$cl(P)$ is the smallest(σ_i, σ_j)-wsI-closed set containing P, therefore we have (σ_i, σ_j)-wsI-$cl(P) \subseteq P \cup \sigma_j$-$int^*(\sigma_i$-$cl(\sigma_j$-$int(P)))$. Hence, (σ_i, σ_j)-wsI-$cl(P) = P \cup \sigma_j$-$int^*(\sigma_i$-$cl(\sigma_j$-$int(P)))$.

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