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Stochastic Analysis to an Power Supply System Through Reliability Modelling

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Abstract. *The increase in demand of an industrial, residential and non-residential world to the power supply system is leading its importance for enhancement of new technologies in the form of new appliances and resources for power deliverance to users. Also according to its supportive and harmful nature, it needs more balanced management process for maintaining its availability, repair and manufacturing mechanism in all environmental conditions. The presented manuscript provides a stochastic analysis of the power supply system based on the model of reliability theory. Emerging reliability model has been investigated for a power supply system operating in two different forms with separate repair facility for varying nature of failures and working process. The inspection process had also been initiated for proper verification of damage and its repairing strategy for complex working system. The reliability modelling is performed in numerical form for explaining profit and reliability behaviour of system with varying nature of repair mechanism and failure rate. The nature of all reliability parameters is explained and concluded with calculated results for betterment and proper organization of power supply system.*

Keywords. Reliability modelling, geometric distribution, system availability, failure cost, inspection cost, maintenance cost, profit function.

Mathematics Subject Classification (2010): 60K10, 60K20, 62N05, 90B25.

1 Introduction

In the real-life involving industries, education, management and social system the common important measure of each is their dependancy on electricity and power system. Electricity is not only the important part of past and present world, but it has it's value in future also. In other words, we cannot assume our daily life, business, industrial system without electricity. So, always there is an race in inventing new technologies and research for its proper functioning and treatment. Being useful in nature, it also had many disadvantages in case of

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any wrong management and it leads to harm the users life too. Hence, we can predict that there must be correct measures to be taken care for the use of the appliances based on electricity and its maintenance too. There are so many examples exist in real world involving electricity or power system. The present paper had initiated its contribution by analyzing the power supply and maintenance system as two different process. We all know that the power supply system to our residential/non-residential systems are followed through two processes a) government power supply connection/system and b) by personal invertor and genset system in case of any failure to first one. In another form, we can say that the second category lies as an warm standby mode for regular functioning of system without any loss of time or cost. In this paper, we had considered the situation by taking two units $'X'$ and $'Y'$ where $'X'$ is the electricity system that states government supply system and $'Y'$ as an warm standby units. Due to complexity of process containing system $'X'$, its maintenance and handling cost along with repair time is more as compared to the second one. So, the repair preference is always counting its value to the unit $'Y'$. Also for unit $'X'$, the inspection policy had also been initiated for proper verification to its damage and repair strategy. Being as a power system, the warm standby unit is always having possibility to get failed in standby mode due to its non-use for long period and also to many other natural reason.

In past times, so many researchers had studied many reliability problems concerning to different technical or non-technical world. In 2008, Bhardwaj et al. [1,2] had stochastically examined distinct repair and failure mechanism under discrete distribution for the redundant system. Evaluation of industrial system with linear first order dierential equations was initiated in 2009 by Haggag et al. [3,4]. Rizwan et al. [5] also investigated hot standby PLC system and Kumar [6] examined a computer based working system with preferable replacement to S/W over H/W. In 2014, Singh et al. [7] had stochastically studied a power generating system of a turbine plant. Malhotra et. al. [8,9], examine system with varying repair demands and Bhatti, Kakkar et al [10–14] initiated with concept of correlation and geometric distributions in reliability.

In 2016, Hua et al. [15,16] had spatially involved unit degradation paths in his research. Pervaiz et al [17] examined paper plant industry using Boolean function but in case of assessing cable plant subsystem S.Z. Taj, et al [18] framed probabilistic modelling. Many mechanical system having assembling and activation time, application of F and G balanced systems under Markov processes are investigated by N. Adlakha [19], Cui et al [20,21] and Endharta et al. [22]. Chen W-L. et al. [23] enhanced his study for retrial machine repair systems with operating units to be in warm standby mode but a single recovery policy for server breakdown. In last year 2019, the study of industrial modeling by Saini [24,25] and Barak [26], neural network prediction model by S. Bhardwaj [27] and use of bivariate wiener processes by Dong Q.L. et al. [28] in reliability had taken the research to large extent. Also balanced mechanisms for examining balanced systems and common cause failures had also been inculcated through Wu H. et al. [29], Jia H.P. [30] and Fang et al. [31]. Bhatti and Kakkar et al. [32–34] also enhance his study under reliability with active or passive standby systems with common failure. As to all above previous research the present paper had also contributed by taking an initiative towards analysis of power system following two mechanism with distinct deliverance, handling and maintenace strategy. Possible states of the system under operative and failed states are reflected through transition model figure. 1.

Availability States

$$A_0 = (X_0, Y_S), \quad A_1 = (X_I, Y_0), \quad A_2 = (X_{r_1}, Y_0), \quad A_5 = (X_0, Y_{r_2}).$$

Failure States

$$F_3 = (X_{r_{1w}}, Y_{r_2}), \qquad F_4 = (X_I, Y_{r_2}).$$

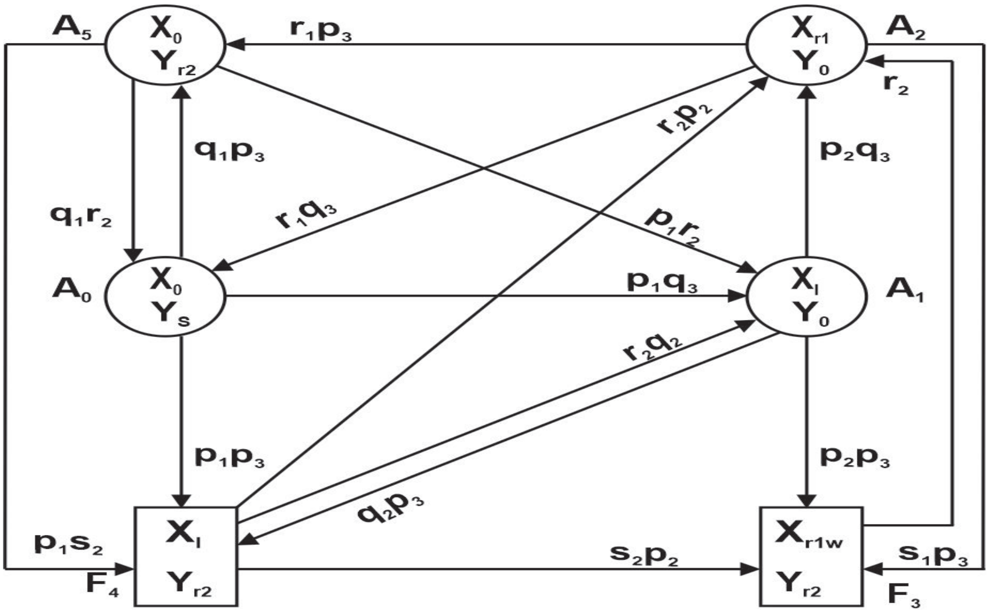

Fig. 1 Transition Model

Table 1 Nomenclature

Symbol		Description
X_0, Y_0	:	Operative behaviour of unit $'X'$ and $'Y'$.
X_I	:	Inspection behaviour of failed unit $'X'$.
Y_S	:	Warm standby behaviour of $'Y'$.
X_{r_1}/Y_{r_2}	:	Repaired service to failure unit $'X'$ and $'Y'$.
p_1	:	Probability value to $'X'$ in failure mode
p_2	:	Probability value to $'X'$ for being better inspected tor its repair.
p_3	:	Probability value to $'Y'$ in failure mode.
r_1/r_2	:	Probability successful repair value to $'X'$ and $'Y'$.

2 Transition Probabilities

Using the transition diagram shown in figure 1, the steady state transition probabilities calculated by applying Q_{ij} depicts the cumulative density function from first regenerative state $'i'$ to second state $'j'$.

$$P_{ij} = \lim_{t\to\infty} Q_{ij}$$

The evaluated transition probabilities are as follows:

$$P_{01} = \frac{p_1 q_3}{1-q_1 q_3}, \quad P_{04} = \frac{p_1 p_3}{1-q_1 q_3}, \quad P_{05} = \frac{q_1 p_3}{1-q_1 q_3}, \quad P_{12} = \frac{p_2 q_3}{1-q_2 q_3},$$

$$P_{13} = \frac{p_2 p_3}{1-q_2 q_3}, \quad P_{14} = \frac{q_2 p_3}{1-q_2 q_3}, \quad P_{20} = \frac{r_1 q_3}{1-s_1 q_3}, \quad P_{23} = \frac{s_1 p_3}{1-s_1 q_3},$$

$$P_{25} = \frac{r_1 p_3}{1-s_1 q_3}, \quad P_{32} = \frac{r_2}{1-s_2}, \quad P_{41} = \frac{r_2 q_2}{1-s_2 q_2}, \quad P_{42} = \frac{r_2 p_2}{1-s_2 q_2},$$

$$P_{43} = \frac{s_2 p_2}{1-s_2 q_2}, \quad P_{50} = \frac{q_1 r_2}{1-q_1 s_2}, \quad P_{51} = \frac{p_1 r_2}{1-q_1 s_2}, \quad P_{54} = \frac{p_1 s_2}{1-q_1 s_2}.$$

21 Mean Sojourn Times

By denoting mentioning sojourn time in state $S_i(i = 0 - 5)$ by symbol $'\mu_i'$, the value of mean sojourn time for state S_i is calculated as:

$$\mu_0 = \frac{1}{1 - q_1 q_3}, \qquad \mu_1 = \frac{1}{1 - q_2 q_3}, \quad \mu_2 = \frac{1}{1 - s_1 q_3},$$
$$\mu_3 = \frac{1}{1 - s_2}, \quad \mu_4 = \frac{1}{1 - s_2 q_2}, \quad \mu_6 = \frac{1}{1 - q_1 s_2}.$$

3 Availability and Maintenance Analysis of the System

Using probabilistic argument and through a graphical description of the model, the relations related to reliability analysis of the system are obtained as equations (1)-(6).

$$X_0 = Z_0 + q_{01} © X_1 + +q_{04} © X_5 + +q_{05} © X_5. \tag{3.1}$$

$$X_1 = Z_1 + q_{12} © X_2 + q_{13} © X_3 + q_{14} © X_4. \tag{3.2}$$

$$X_2 = Z_2 + q_{20} © X_0 + q_{23} © X_3 + q_{25} © X_5. \tag{3.3}$$

$$X_3 = Z_3 + q_{32} © X_2. \tag{3.4}$$

$$X_4 = Z_4 + q_{41} © X_1 + q_{42} © X_2 + q_{43} © X_3. \tag{3.5}$$

$$X_5 = Z_5 + q_{50} © X_0 + q_{51} © X_1 + q_{54} © X_4. \tag{3.6}$$

By solving this equations, value of some reliability parameters as mean time to system failure, availability, busy schedule of repairman r_1, busy schedule of repairman r_2 are directly assessed.

Mean Time to System Failure (MTSF):

$$MTSF = \frac{N_1}{D_1}$$

Availability:

$$A_0 = -\frac{N_2(1)}{D_2'(1)}, \qquad Z_i = 0 \qquad for i = 3, 4.$$

Busy schedule of Inspection:

$$B_0 = -\frac{N_3(1)}{D_2'(1)}, \qquad Z_i = 0 \qquad for i = 0, 2, 3, 5.$$

Busy schedule of Repairman r_1:

$$B_0' = -\frac{N_4(1)}{D_2'(1)}, \qquad Z_i = 0 \qquad for i = 0, 1, 3, 4, 5.$$

Busy schedule of Repairman r_2:

$$B_0'' = -\frac{N_5(1)}{D_2'(1)}, \qquad Z_i = 0 \qquad for i = 0, 1, 2.$$

where

$$\begin{aligned} N_1 &= \mu_0 - 1 + \mu_1(P_{01} + P_{05}P_{51}) + P_{12}[\mu_2(P_{01} + P_{05}P_{51}) + P_{20}] \\ &\quad + P_{01}P_{12}P_{25}[\mu_5 + P_{50} + P_{51}(1 - \mu_0)] + \mu_5 P_{05} \end{aligned} \tag{3.7}$$

$$D_1 = 1 - P_{01}P_{20}P_{12} - P_{12}P_{25}(P_{51} + P_{50}P_{01}) - P_{50} - P_{12}P_{20}P_{51} \tag{3.8}$$

$$\begin{aligned} N_2(1) &= \mu_0(P_{20} - P_{25}P_{50})(1 - P_{14}P_{41}) + P_{01}[\mu_1(1 - P_{23} - P_{25}P_{54}(1 - P_{41})) \\ &\quad + \mu_2(1 - P_{14}P_{41}) + \mu_5 P_{25}(1 - P_{14}P_{41})] - P_{04}[(P_{42} + P_{43}P_{32}) \\ &\quad (\mu_1 P_{25}P_{51} + \mu_2 + \mu_2 P_{25}P_{41}) + (P_{12} + P_{13}P_{32})(\mu_2 P_{41} + \mu_5 P_{25}P_{41}) \\ &\quad + \mu_1 P_{41}(1 - P_{23}P_{32})] + P_{05}[(P_{51} + P_{54}P_{41})(\mu_1(1 - P_{23}P_{32}) \\ &\quad - \mu_2(P_{12} + P_{13}P_{32})) - (P_{42} + P_{43}P_{32})(\mu_2(P_{54} + P_{51}P_{14}) \\ &\quad + \mu_5(1 - P_{23}P_{32})(1 - P_{14}P_{41}))] \end{aligned} \tag{3.9}$$

$$\begin{aligned} N_3(1) &= P_{01}[\mu_1(1 - P_{23} - P_{25}P_{54}(1 - P_{41})) + \mu_4(P_{25}P_{54}(1 - P_{14}) \\ &\quad + P_{14}(1 - P_{23}))] - P_{04}[(1 - P_{23}P_{32})(\mu_1 P_{41} + \mu_4) + P_{25}P_{51}(\mu_1(P_{42} \\ &\quad + P_{43}P_{32}) + \mu_4(P_{13} + P_{12}P_{23}))] + P_{05}[(1 - P_{23}P_{32}) \\ &\quad [\mu_1(P_{51} + P_{54}P_{41}) + \mu_4(P_{54} + P_{51}P_{14})] \end{aligned} \tag{3.10}$$

$$\begin{aligned} N_4(1) &= P_{01}\mu_2(1 - P_{14}P_{41}) + P_{04}\mu_2[(P_{42} + P_{43}P_{32}) + P_{41}(P_{12} + P_{13}P_{32})] \\ &\quad + P_{05}\mu_2[(P_{42} + P_{43}P_{32})(P_{54} + P_{51}P_{14}) + (P_{12} + P_{13}P_{32}) \\ &\quad (P_{51} + P_{54}P_{41})]. \end{aligned} \tag{3.11}$$

$$\begin{aligned} N_5(1) &= P_{01}[\mu_3(P_{23}(P_{12} + P_{13}P_{32}) + (P_{13} + P_{12}P_{23}) + P_{25}P_{54}(P_{12}P_{43} \\ &\quad - P_{13}P_{42})) + \mu_4(P_{25}P_{54}(1 - P_{14}) + P_{14}(1 - P_{23})) + \mu_5 P_{25} \\ &\quad (1 - P_{14}P_{41})] + P_{04}[\mu_3((P_{43} + P_{42}P_{23}) + P_{41}(P_{13} + P_{12}P_{23}) + P_{25}P_{51} \\ &\quad (P_{12}P_{43} - P_{13}P_{42})) + \mu_4((1 - P_{23}P_{32}) - P_{25}P_{51}(P_{12} + P_{13}P_{32})) \\ &\quad + \mu_5 P_{25}((P_{42} + P_{43}P_{32}) + P_{41}(P_{12} + P_{13}P_{32}))] + P_{05}[\mu_3((P_{54} \\ &\quad + P_{51}P_{14})(P_{43} + P_{42}P_{23}) + (P_{13} + P_{12}P_{23})(P_{51} + P_{54}P_{41})) \\ &\quad + \mu_4(1 - P_{23}P_{32})(P_{54} + P_{51}P_{14}) + \mu_5(1 - P_{23}P_{32})(1 - P_{41}P_{14})] \end{aligned} \tag{3.12}$$

$$\begin{aligned} D_2'(1) &= -[\mu_0(P_{20} - P_{25}P_{50})(1 - P_{14}P_{41}) + P_{01}[\mu_1(1 - P_{23} - P_{25}P_{54} \\ &\quad (1 - P_{41})) + \mu_2(1 - P_{14}P_{41}) + \mu_3(P_{23}(P_{12} + P_{13}P_{32}) \\ &\quad + (P_{13} + P_{12}P_{23}) + P_{25}P_{54}(P_{12}P_{43} - P_{13}P_{42})) + \mu_4(P_{25}P_{54}(1 - P_{14}) \\ &\quad + P_{14}(1 - P_{23})) + \mu_5 P_{25}(1 - P_{14}P_{41})] + P_{04}[\mu_1 P_{41}(1 - P_{23}P_{32}) \\ &\quad + P_{25}P_{51}\mu_1(P_{42} + P_{43}P_{32}) + \mu_2[(P_{42} + P_{43}P_{32}) + P_{41}(P_{12} + P_{13}P_{32})] \\ &\quad + \mu_3((P_{43} + P_{42}P_{23}) + P_{41}(P_{13} + P_{12}P_{23}) + P_{25}P_{51}(P_{12}P_{43} - P_{13}P_{42})) \\ &\quad + \mu_4((1 - P_{23}P_{32}) - P_{25}P_{51}(P_{12} + P_{13}P_{32})) + \mu_5 P_{25}((P_{42} + P_{43}P_{32}) \\ &\quad + P_{41}(P_{12} + P_{13}P_{32}))] + P_{05}[\mu_1(1 - P_{23}P_{32})(P_{51} + P_{54}P_{41}) \\ &\quad + \mu_2[(P_{42} + P_{43}P_{32})(P_{54} + P_{51}P_{14}) + (P_{12} + P_{13}P_{32})(P_{51} + P_{54}P_{41})] \\ &\quad + \mu_3((P_{54} + P_{51}P_{14})(P_{43} + P_{42}P_{23}) + (P_{13} + P_{12}P_{23})(P_{51} + P_{54}P_{41})) \\ &\quad + \mu_4(1 - P_{23}P_{32})(P_{54} + P_{51}P_{14}) + \mu_5(1 - P_{23}P_{32})(1 - P_{41}P_{14})]] \end{aligned} \tag{3.13}$$

4 Conclusion

The total profit of system calculated as

$$P = C_1 A_0 - C_2 B_0 - C_3 [B_0' + B_0'']$$

by using C_1 the per unit up time revenue by the system, C_2, C_3 the per unit down time expenditure on the system.

Through the data analysis, the income function P was calculated for the specific value of the parameters and the results were analyzed.

$C_0 = 5000$, $C_1 = 500$, $C_2 = 800$, $C_3 = 300$ and $p_2 = 0.8$.

Table 2 reflects that the profit function will decrease with respect to failure rate p_1 for certain r_1, r_2, p_2 values. Whereas Table 3 reflects its increasing behaviour for certain p_1, p_2 with increasing r_1, r_2.
Hence, with the help of numerical analysis it has been proved that the profit function increases with increasing repair and decreasing failure rate. The growing demand for energy systems in modern time creates a need for the expansion of new technologies. Researches in the field of energy supply system analysis are particular importance. In this regard, the issue discussed in the article taking interest. Determining the total profit of system and reliability of two power supply systems with different repair methods and causes of failure deserves positive attention.

Table 2 Reliability parameters with respect to repair r_1,r_2, failure Rate p_2.

Repair, Failure Rate	MTSF	A_0	B_0	B_0'	B_0''	Profit
r_1 = 0.1, r_2 = 0.05, p_2 = 0.4	6.236263041	0.215278	0.186736	0.155135	0.828634	610.3243101
	3.660700618	0.206024	0.187166	0.154309	0.830791	563.8535295
	2.709070971	0.196896	0.188633	0.151187	0.8304	520.0922004
	2.175566762	0.186919	0.190831	0.146228	0.828317	473.704228
	1.820467635	0.175319	0.193785	0.139213	0.824598	420.9506832
	1.562532265	0.161097	0.197713	0.12945	0.818873	357.40438
	1.365989041	0.142649	0.203075	0.115583	0.81028	276.1581849
	1.212355688	0.116993	0.210789	0.094938	0.797051	164.5029402
r_1 = 0.08, r_2 = 0.07, p_2 = 0.2	8.699514715	0.165298	0.187539	0.147359	0.791224	377.4676578
	4.555432704	0.163116	0.191886	0.154762	0.808877	353.1660556
	3.108715243	0.159434	0.19829	0.15711	0.824097	325.1070359
	2.353492043	0.15466	0.206892	0.156532	0.839848	292.6726629
	1.883675731	0.148606	0.218449	0.153498	0.858228	253.5373715
	1.561417235	0.140682	0.234488	0.147656	0.881787	203.5048548
	1.326541872	0.129647	0.258041	0.13772	0.914849	134.5789649
	1.148442541	0.112707	0.295867	0.120497	0.966594	29.22459592
r_1 = 0.12, r_2 = 0.03, p_2 = 0.1	12.28963229	0.147405	0.188143	0.144881	0.785851	291.2923054
	5.603605951	0.146394	0.194898	0.156512	0.808535	266.7507065
	3.528805358	0.143746	0.203987	0.161651	0.829288	238.6265307
	2.535744442	0.140123	0.21592	0.163191	0.851597	206.6216231
	1.956997706	0.135447	0.231933	0.161967	0.878403	168.1726739
	1.57922852	0.129253	0.254377	0.157829	0.913682	118.7061321
	1.313970809	0.12049	0.287996	0.149614	0.964641	49.36804227
	1.118153844	0.106658	0.343847	0.134057	1.047549	-60.14194741

Table 3 Reliability parameters w.r.t Failure rate p_1, p_2.

Failure Rate p_1, p_2	MTSF	A_0	B_0	B_0'	B_0''	Profit
$p_1 = 0.9$ $p_2 = 0.4$	1.092772452	0.16627	0.071238	0.125439	0.845021	441.8753032
	1.115773209	0.240059	0.096239	0.167054	0.869074	757.8114202
	1.141174572	0.299294	0.107093	0.19196	0.867498	1029.106808
	1.169030975	0.349753	0.107102	0.206959	0.849098	1274.919214
	1.199426379	0.394572	0.099024	0.215873	0.8193	1504.857375
	1.232474237	0.435555	0.085041	0.220963	0.781576	1724.009566
	1.268318428	0.4738	0.066842	0.223616	0.738208	1935.225573
	1.30713512	0.51001	0.045719	0.22471	0.690715	2140.209512
$p_1 = 0.7$ $p_2 = 0.2$	1.327907017	0.186837	0.220818	0.174372	1.12046	348.1406109
	1.390929928	0.243734	0.276046	0.188599	1.117986	594.3718414
	1.455891278	0.2882	0.310699	0.187741	1.107788	803.1237968
	1.522920841	0.327568	0.3363	0.182581	1.093774	995.4944972
	1.592157195	0.364518	0.358172	0.176693	1.077425	1178.920274
	1.663748955	0.400145	0.379254	0.17134	1.059405	1356.206244
	1.737856097	0.434904	0.401364	0.166933	1.040045	1528.276156
	1.814651415	0.468945	0.425723	0.163539	1.019524	1695.174165
$p_1 = 0.4$ $p_1 = 0.1$	2.5369863	0.164197	0.394377	0.261439	1.338223	13.17785063
	2.856205424	0.200478	0.449298	0.218345	1.28709	216.9389394
	3.179431921	0.232195	0.490993	0.180217	1.257472	394.0621232
	3.507411179	0.262616	0.530822	0.151437	1.236987	555.4240895
	3.840869315	0.292685	0.572781	0.130799	1.221735	705.8753844
	4.18052332	0.322755	0.618756	0.11659	1.210285	848.0418257
	4.527090061	0.352972	0.669816	0.107365	1.202159	983.4101784
	4.881294474	0.383376	0.726653	0.102015	1.197317	1112.749464

References

1. Bhardwaj, N., Kumar, A., Kumar, S.: *Stochastic analysis of a single unit redundant system with two kinds of failure and repairs*, Reections des. ERA-JMS, **3(2)**, 115-134 (2008,).
2. Bhardwaj, N.: *Analysis of two-unit redundant system with imperfect switching and connection time*, International Transactions in Mathematical Sciences and Computer, **2(2)**, 95-202 (2009).
3. Haggag, M.Y.: *Cost analysis of a system iInvolving common cause failures and preventive maintenance*, J. Math and Stat., **5(4)**, 305-310 (2009).
4. Haggag, M.Y.: *Cost analysis of two-dissimilar unit cold standby system with three states and preventive maintenance using linear rst order dierential equations*, J. Math and Stat., **5(4)**, 395-400 (2009).
5. Rizwan, S.M., Khurana, V., Taneja, G.: *Reliability analysis of a hot standby industrial system*, International Journal of Modelling and Simulation, **30(3)**, 315-322 (2010).
6. Kumar, A., Malik, S.C.: *Reliability modeling of a computer system with priority to S/w replacement over H/w replacement subject to MOT and MRT*, International Journal of Pure and Applied Math-ematics, **80(5)**, 693–709 (2012).
7. Singh, D., Taneja, G.: *Reliability and economic analysis of a power generating system comprising one gas and one steam turbine with random inspection*, Journal of Mathematics and Statistics, **10(4)**, 436–442 (2014).
8. Malhotra, R., Taneja, G.: *Stochastic analysis of a two-unit cold standby system wherein both units may become operative depending upon the demand*, Journal of Quality and Reliability Engineering, 1-13 (2014).
9. Malhotra, R., Taneja, G.: *Comparative study between a single unit system and a two unit cold standby system withvarying demand*, SpringerPlus, **4**, 1–17 (2015).
10. Bhatti, J., Chitkara, A., Kakkar, M.: *Stochastic analysis of parallel system with two discrete failures*, Model Assisted Statistics and Applications, **9**, 257–265 (2014).

11. Kakkar, M. K., Bhatti, J.: *Reliability and profit analysis of standby unit system with correlated life time in an industry*, Advance Study in Contemporary Mathematics, **25(3)**, 333-340 (2015).
12. Kakkar, M. K., Chitkara, A. K., Bhatti, J.: *Reliability analysis of two-unit parallel repairable industrial system*, Decision Science Letters, **4**, 525-536, (2015).
13. Kakkar, M. K., Chitkara, A. K., Bhatti, J.: *Reliability analysis of two dissimilar parallel unit repairable system with failure during preventive maintenance*, Management Science Letters, **6**, 285–296 (2016).
14. Bhatti, J., Chitkara, A., Kakkar, M.: *Stochastic analysis of dis-similar standby system with discrete failure*, inspection and replacement policy, Demonstratio Mathematica, **49(2)**, 224-235 (2016).
15. Hua, D.G., Elsayed, E.: *Reliability estimation of k-out-of-npairs: G balanced systems with spatially distributed units*, IEEE Trans. Reliab., **65**, 886-900 (2016).
16. Hua, D.G., Elsayed, E.: *Degradation analysis of k-out-of-n pairs: G balanced systems with spatially distributed units*, IEEE Trans Reliab., **65**, 941-56 (2016).
17. Iqbal, P., Uduman, P.S.S.: *Reliability analysis of paper plant using boolean function with fuzzy logic technique*, International Journal of Applied Engineering Research, **11(1)**, 573-577 (2016).
18. Taj, S.Z., Rizwan, S.M., Alkali, B.M., Harrison, D.K., Taneja, G.: *Probabilistic modeling and analysis of a cable plant subsystem with priority to repair over preventive maintenance*, I-Managers Journal on Mathematics, **6(3)**, 12-21 (2017).
19. Adlakha, N., Taneja, G., Shilpi: *Reliability and cost-benet analysis of a two-unit cold standby system used for communication through satellite with assembling and activation time*, International Journal of Applied Engineering Research, **12(20)**, 9697-9702 (2017).
20. Cui, L.R., Gao, H.D., Mo, Y.C.: *Reliabilities for k-out-of-n: F balanced systems with m sectors*, IISE Trans., **50(5)**, 381-393 (2017).
21. Cui, L.R., Chen, J.H., Li, X.C.: *Balanced reliability systems under Markov processes*, IISE Trans., **51(9)**, 1025-35 (2018).
22. Endharta, A.J., Yun, W.Y., Ko, Y.M.: *Reliability evaluation of circular k-out-of-n:G balanced systems through minimal path sets*, Reliability Engineering and System Safety, **180**, 220-236 (2018).
23. Chen, W-L.: *System reliability analysis of retrial machine repair systems with warm standbys and a single server of working breakdown and recovery policy*, System Engineering, **21**, 59-69 (2018).
24. Saini, M., Kumar, A.: *Stochastic modeling of a single-unit system operating under different environmental conditions subject to inspection and degradation*, Proc. Natl. Acad. Sci., India, Sect. A Phys. Sci., **21**, 1-8 (2018).
25. Kumar, A., Saini, M.: *Analysis of some reliability measures of single-unit systems subject to abnormal environmental conditions and arbitrary distribution for failure and repair activities*, Journal of Information and Optimization Sciences, **39(2)**, 545559 (2018).
26. Barak, M.S., Yadav, D., Barak, S.K.: *Stochastic analysis oftwo unit redundant system with priority toinspection overrepair*, Life Cycle Reliability and Safety Engineering, **21**, 1-9 (2018).
27. Bhardwaj, S., Bhardwaj, N., Kumar, V., Parashar, B.: *Estimation of lifespan of diesel locomotive engine*, Journal of Information and Optimization Sciences, **40(5)**, 1097–1108 (2019).
28. Dong, Q.L, Cui, L.R., Si, S.B.: *Reliability and availability analysis of stochastic degradation systems based on bivariate wiener processes*, Appl. Math. Model, **79**, 414-433 (2019).

29. Wu, H., Li, Y.F., Brenguer, C.: *Optimal inspection and maintenance for a repairable k -out-of-n: G warm standby system*, Reliability Engineering and System Safety, **193**, 1–11 (2019).
30. Jia, H.P., Ding, Y., Peng, R., Liu, H.L., Song, Y.H.: *Reliability assessment and activation sequence optimization of non-repairable multi-state generation systems considering warm standby*, Reliability Engineering and System Safety, **195**, 1–10 (2019).
31. Fang, C., Cui, L.: *Reliability analysis for balanced engine systems with m sectors by considering start-up probability*, Reliability Engineering and System Safety, **197**, 1–10 (2020).
32. Kakkar, M.K., Bhatti, J., Malhotra, R., Kaur, M., Goyal, D.: *Availability analysis of an industrial system under the provision of replacement of a unit using genetic algorithm*, International Journal of Innovative Technology and Exploring Engineering (IJITEE), **9**, 1236–1241 (2019).
33. Bhatti, J., Kakkar, M.K., Bhardwaj, N., Kaur, M., Goyal D.: *Reliability analysis to industrial active standby redundant system*, Malaysian Journal of Science, **39(3)**, 74–84, (2020).
34. Bhatti, J., Kakkar, M.K.: *Reliability analysis of cold standby parallel system possessing failure and repair rate under geometric distribution*, Recent Advances in Computer Science and Communications, **13**, 1–7 (2020).