Trans. Natl. Acad. Sci. Azerb. Ser. Phys.-Tech. Math. Sci.
Mathematics, **41** (1), 1-13 (2021).
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Introduction to Fuzzy Topology on Soft Sets

Cigdem Gunduz Aras ⋆ · **Sadi Bayramov** · **Kemale Veliyeva**

Received: 12.07.2020 / Revised: 06.12.2020 / Accepted: 26.01.2021

Abstract. *The purpose of the paper is to define gradation of openness on soft sets τ and examine some of its important properties. For this, we give some fundemental properties of soft sets. Later we introduce the concepts of base and subbase in fuzzy topological space of soft sets, and use them to discuss continuous mapping and open mapping.*

Keywords. Soft set, gradation of openness, base and subbase in fuzzy topological space.

Mathematics Subject Classification (2010): 54A40, 54A05, 06D72

1 INTRODUCTION

The concept of fuzzy sets was introduced by Zadeh in his sounding article [20]. Fuzzy sets help for generalizing many of the concept of usual topology which may be named fuzzy topological spaces. After the idea of fuzzy theory into topology was given by Chang [4], this theory has been studied and applied in a miscellaneous areas. So they also have discussed different aspects of fuzzy topology. Sostak initiated fuzzy gradation of openness on fuzzy subsets of a nonempty set X in [17]. Yue and Fang extended Lowen functors to $I-$fuzzy topological spaces in [19]. Y. Yue gave $LM-$fuzzy topological spaces in [18] and studied the stratifications of $LM-$fuzzy topologies. As in other theories, for example rough sets [15], vague sets [5] etc., all these theories have their inherent difficulties. In 1999, D. Molodtsov [13] introduced the concept of soft set theory which is a completely new approach for modeling uncertainty. Thus he aimed to describe phenomena and concepts of an ambiguous, vague, undefined and imprecise meaning. Since soft set theory has a rich

⋆ Corresponding author

C. Gunduz Aras
Department of Mathematics, Kocaeli University, Kocaeli, 41380-Turkey
E-mail: carasgunduz@gmail.com

S. Bayramov
Department of Algebra and Geometry, Baku State Universiy, Baku Az1148, Azerbaican
E-mail: baysadi@gmail.com

K. Veliyeva
Department of Algebra and Geometry, Baku State Universiy, Baku Az1148, Azerbaican
E-mail: kemale2607@mail.ru

potential, researches on soft set theory and its applications in various fields are progressing rapidly in [10], [11]. The applications of soft set theory in algebraic structures was introduced by Aktas and Cagman [1]. They introduced soft groups and investigated some basic properties and compared soft sets to fuzzy and rough sets. C. Gunduz(Aras) and S. Bayramov [6,7] introduced fuzzy soft modules and intuitionistic fuzzy soft modules and investigated some fundemental properties.

Topological stuructures of soft set have been studied by some authors in recent years. M. Shabir, M. Naz [16] have initiated the study of soft topological spaces which are defined over an initial universe with a fixed set of parameters and showed that a soft topological space gives a parameterized family of topological spaces. Undoubtedly, soft topological spaces are an important generalization of topological spaces. It is observed in the last few years that a large number of papers was devoted to the study of soft topological spaces in [2,3,8,9,12,14].

In this paper, we give the definition of gradation of openness τ which is a mapping from $SS(X,E)$ to $[0,1]$ which satisfies some definite conditions. Then we show that a fuzzy topological space gives a parameterized family of soft topologies on X. Later we introduce the concepts of base and subbase in fuzzy topological space of soft sets, and use them to discuss continuous mapping and open mapping.

2 PRELİMİNARY

In this section we will introduce necessary definitions and theorems for soft sets. Throughout this paper, X refers an initial universe, E is the set of all parameters, $SS(X,E)$ denotes the family of all soft sets over X with a fixed set of parameters E.

Definition 2.1 *[13] A pair (F,A) is called a soft set over X, where F is a mapping given by $F : A \to P(X)$.*

In other words, the soft set is a parameterized family of subsets of the set X. For $e \in A$, $F(e)$ may be considered as the set of $e-$elements of the soft set (F,A), or as the set of $e-$approximate elements of the soft set, i.e.,

$$(F,A) = \{(e, F(e)) : e \in A \subseteq E,\ F : A \to P(X)\}.$$

Definition 2.2 *[11] The intersection of two soft sets (F,A) and (G,B) over X is the soft set (H,C), where $C = A \cap B$ and $\forall e \in C$, $H(e) = F(e) \cap G(e)$. This is denoted by $(F,A)\widetilde{\cap}(G,B) = (H,C)$.*

Definition 2.3 *[11] The union of two soft sets (F,A) and (G,B) over X is the soft set, where $C = A \cup B$ and $\forall e \in C$,*

$$H(e) = \begin{cases} F(e), & \text{if } e \in A - B, \\ G(e), & \text{if } e \in B - A, \\ F(e) \cup G(e) & \text{if } e \in A \cap B. \end{cases}$$

This relationship is denoted by $(F,A)\widetilde{\cup}(G,B) = (H,C)$.

Definition 2.4 *[16] A soft set (F,E) over X is said to be a null soft set, denoted by Φ, if $F(e) = \varnothing$ for all $e \in E$.*

Definition 2.5 *[16] A soft set (F,E) over X is said to be an absolute soft set, denoted by $\widetilde{X}$, if $F(e) = X$ for all $e \in E$.*

Definition 2.6 *[16] The difference (H,E) of two soft sets (F,E) and (G,E) over X, denoted by $(F,E)\backslash(G,E)$, is defined as $H(e) = F(e)\backslash G(e)$ for all $e \in E$.*

Definition 2.7 *[16] The complement of a soft set (F, E), denoted by $(F, E)^c$, is defined $(F, E)^c = (F^c, E)$, where $F^c : E \to P(X)$ is a mapping given by $F^c(e) = X \backslash F(e)$ for all $e \in E$ and F^c is called the soft complement function of F.*

Definition 2.8 *[9] Let (X, E) and (Y, E') be two soft sets, $f : X \to Y$ and $g : E \to E'$ be two mappings. Then $(f_g) : (X, E) \to (Y, E')$ is called a soft mapping and defined as: for a soft set (F, A) in (X, E), $(f_g)((F, A)) = f(F)_{g(a)}$, $B = g(A) \subseteq E'$ is a soft set in (Y, E') given by*

$$f(F)(e') = \begin{cases} f\left(\underset{e\in g^{-1}(e')\cap A}{\cup} F(e)\right), & \text{if } g^{-1}(e') \cap A \neq \emptyset, \\ \emptyset, & \text{otherwise,} \end{cases}$$

for $e' \in B \subseteq E'$. $(f(F), g(A))$ is called a soft image of a soft set (F, A).

Definition 2.9 *[9] Let (X, E) and (Y, E') be two soft sets, $(f_g) : (X, E) \to (Y, E')$ be a soft mapping and $(G, C)\widetilde{\subseteq}(Y, E')$. Then $(f_g)^{-1}((G, C)) = f^{-1}(G)_{g^{-1}(C)}$ is a soft set in the soft set (X, E) and $D = g^{-1}(C)$, defined as:*

$$f^{-1}(G)(e) = \begin{cases} f^{-1}(G(g(e))), & \text{if } g(e) \in C, \\ \emptyset, & \text{otherwise,} \end{cases}$$

for $e \in D \subseteq E$. $(f_g)^{-1}((G, C))$ is called a soft inverse image of (G, C).

Definition 2.10 *[16] Let τ be the collection of soft set over X, then τ is said to be a soft topology on X if*

1) $\Phi, \widetilde{X}$ belong to τ;
2) the union of any number of soft sets in τ belongs to τ;
3) the intersection of any two soft sets in τ belongs to τ.
The triplet (X, τ, E) is called a soft topological space over X.

Definition 2.11 *[16] Let (X, τ, E) be a soft topological space over X. Then members of τ are said to be soft open sets in X.*

Definition 2.12 *[16] Let (X, τ, E) be a soft topological space over X. A soft set (F, E) over X is said to be a soft closed in X if its complement $(F, E)^c$ belongs to τ.*

Definition 2.13 *[2] Let (F, E) be a soft set over X. The soft set (F, E) is called a soft point, denoted by (x_e, E), if for the element $e \in E$, $F(e) = \{x\}$ and $F(e') = \varnothing$ for all $e' \in E - \{e\}$ (briefly denoted by x_e).*

3 Introduction to Fuzzy Topology on Soft Sets

Definition 3.1 *A mapping $\tau : SS(X, E) \to [0, 1]$ is called a gradation of openness of soft sets on X if it satisfies the following conditions:*

(i) $\tau(\Phi) = \tau\left(\widetilde{X}\right) = 1$,
(ii) $\tau\left((F, E)\widetilde{\cap}(G, E)\right) \geq \tau(F, E) \wedge \tau(G, E), \forall (F, E), (G, E) \in SS(X, E)$,
(iii) $\tau\left(\underset{i\in\Delta}{\cup}(F_i, E)\right) \geq \underset{i\in\Delta}{\wedge}\tau(F_i, E), \forall (F_i, E) \in SS(X, E), i \in \Delta$.
The triple (X, E, τ) is called fuzzy topological space of soft sets and denoted by FTS.

Definition 3.2 *A mapping $\upsilon : SS(X,E) \to [0,1]$ is called a gradation of closedness of soft sets on X if it satisfies the following conditions:*

(i$'$) $\upsilon(\Phi) = \upsilon\left(\widetilde{X}\right) = 1,$

(ii$'$) $\upsilon\left((F,E)\,\widetilde{\cup}(G,E)\right) \geq \upsilon(F,E) \wedge \upsilon(G,E),$

(iii$'$) $\upsilon\left(\underset{i\in\Delta}{\cap}(F_i,E)\right) \geq \underset{i\in\Delta}{\wedge}\upsilon(F_i,E), \forall (F_i,E) \in SS(X,E), i \in \Delta.$

The triple (X,E,υ) is called fuzzy cotopological space of soft sets and denoted by $FCTS$.

Theorem 3.1 *a) If τ is a gradation of openness on X, then υ is a gradation of closedness on X such that $\upsilon(F,E) = \tau((F,E)^c)$.*

b) If υ is a a gradation of closedness on X, then τ is a gradation of openness on X such that $\tau(F,E) = \upsilon((F,E)^c)$.

Proof. It is clear that

$\upsilon(\Phi) = \tau(\Phi^c) = \tau\left(\widetilde{X}\right) = 1, \upsilon\left(\widetilde{X}\right) = \tau\left(\widetilde{X}^c\right) = \tau(\Phi) = 1,$

$\upsilon\left((F,E)\,\widetilde{\cup}(G,E)\right) = \tau\left(\left((F,E)\,\widetilde{\cup}(G,E)\right)^c\right) = \tau\left((F,E)^c\,\widetilde{\cap}(G,E)^c\right) \geq \tau((F,E)^c)\wedge$ $\tau((G,E)^c) = \upsilon(F,E) \wedge \upsilon(G,E),$

and

$\upsilon\left(\underset{i\in\Delta}{\cap}(F_i,E)\right) = \tau\left(\left(\underset{i\in\Delta}{\cap}(F_i,E)\right)^c\right) = \tau\left(\underset{i\in\Delta}{\cup}(F_i,E)^c\right) \geq \underset{i\in\Delta}{\wedge}\tau(F_i,E)^c = \underset{i\in\Delta}{\wedge}\upsilon(F_i,E).$

The proof is completed.

b) The proof is done similiar to a).

Theorem 3.2 *Let (X,E,τ) be FTS. Then for each $r \in (0,1]$,*

$$\tau_r = \{(F,E) \in SS(X,E) :\ \tau(F,E) \geq r\}$$

is descending family of soft topologies of soft sets on X .

Proof. Since $\tau(\Phi) = \tau\left(\widetilde{X}\right) = 1 \geq r$, then $\Phi, \widetilde{X} \in \tau_r$. If $(F,E),(G,E) \in \tau_r$, $\tau\left((F,E)\,\widetilde{\cap}(G,E)\right) \geq$ $\tau(F,E) \wedge \tau(G,E) \geq r$. Hence $(F,E)\,\widetilde{\cap}(G,E) \in \tau_r$. If $(F_i,E) \in \tau_r$, for $i \in \Delta$, $\tau\left(\underset{i\in\Delta}{\cup}(F_i,E)\right) \geq \underset{i\in\Delta}{\wedge}\tau(F_i,E) \geq r$. Then $\underset{i\in\Delta}{\cup}(F_i,E) \in \tau_r$. So τ_r is a soft topology for each $r \in (0,1]$.

It is obvious that $\{\tau_r\}_{r\in(0,1]}$ is a descending family of soft topologies on soft sets on X.

Remark 3.1 Let (X,E,τ) be a FTS. Then for each $r \in (0,1]$, a fuzzy topological space gives a parameterized family of soft topological spaces on X.

Theorem 3.3 *Let $\{\gamma_r\}_{r\in(0,1]}$ be a descending family of soft topologies on X. Then*

$$\tau(F,E) = \vee\{r : (F,E) \in \gamma_r\},$$

is gradation of openness. Also, $\tau_r = \gamma_r$.

Proof. Since $\Phi, \widetilde{X} \in \gamma_r$, for each $r \in (0,1]$, $\tau(\Phi) = \tau\left(\widetilde{X}\right) = 1$ hold. Next let $(F,E),(G,E) \in SS(X,E)$, $\tau(F,E) = r_1$, $\tau(G,E) = r_2$ and $r = \min\{r_1, r_2\}$. If $r = 0$, then obviously $\tau\left((F,E)\widetilde{\cap}(G,E)\right) \geq 0 = \tau(F,E) \wedge \tau(G,E)$. Suppose that $r > 0$. Choose $\varepsilon > 0$ such that $0 < r - \varepsilon < r$. Then we choose $t_1, t_2 \in (0,1)$ such that $r_1 - \varepsilon < t_1$, $r_2 - \varepsilon < t_2$ and $(F,E) \in \gamma_{t_1}$, $(G,E) \in \gamma_{t_2}$. Let $t = \min\{t_1, t_2\}$. Then $(F,E),(G,E) \in \gamma_t$ (since $\{\gamma_r\}_{r \in (0,1]}$ be a descending family). Hence $(F,E),(G,E) \in \gamma_t$ (since γ_t is a soft topology). So, $\tau\left((F,E)\widetilde{\cap}(G,E)\right) \geq t \geq r - \varepsilon$. Since $\varepsilon > 0$ is arbitrary, therefore $\tau\left((F,E)\widetilde{\cap}(G,E)\right) \geq r = \tau(F,E) \wedge \tau(G,E)$.

Let $\{(F_i,E)\}_{i \in \Delta}$ be a family of soft sets, $P_i = \tau(F_i,E), i \in \Delta$ and $P = \underset{i \in \Delta}{\wedge} P_i$.If $P = 0$, then obviously,

$$\tau\left(\underset{i \in \Delta}{\cup}(F_i,E)\right) \geq 0 = \underset{i \in \Delta}{\wedge}\tau(F_i,E).$$

If $P > 0$, choose $\varepsilon > 0$ such that $P - \varepsilon > 0$. For $i \in \Delta$, $\tau(F_i,E) \geq P > P - \varepsilon$. So there exists γ_r such that $(F_i,E) \in \gamma_r$ and $r \geq P - \varepsilon$. Since γ_r is a soft topology, $\underset{i \in \Delta}{\cup}(F_i,E) \in \gamma_r$. So

$$\tau\left(\underset{i \in \Delta}{\cup}(F_i,E)\right) \geq r > P - \varepsilon.$$

Since $\varepsilon > 0$ is arbitrary,

$$\tau\left(\underset{i \in \Delta}{\cup}(F_i,E)\right) \geq P = \underset{i \in \Delta}{\wedge}\tau(F_i,E).$$

Definition 3.3 *Let (X,E,τ) be a FTS.*

a) $\beta : SS(X,E) \to [0,1]$ is called a base of τ if β satisfies the following condition:

$$\forall (F,E) \in SS(X,E),\ \tau(F,E) = \underset{\underset{i \in \Delta}{\cup}(G_i,E)=(F,E)}{\vee}\ \underset{i \in \Delta}{\wedge}\beta(G_i,E)$$

b) $\varphi : SS(X,E) \to [0,1]$ is called a subbase of τ if $\widetilde{\varphi} : SS(X,E) \to [0,1]$ is a base of τ, where

$$\widetilde{\varphi}(F,E) = \underset{\underset{j \in J}{\cap}(G_j,E)=(F,E)}{\vee}\ \underset{j \in J}{\wedge}\varphi(G_j,E),$$

and J is finite set.

Theorem 3.4 *If $\beta : SS(X,E) \to [0,1]$ satisfies the following conditions,*

a) $\beta(\Phi) = \beta\left(\widetilde{X}\right) = 1$,
b) $\beta\left((F,E)\widetilde{\cap}(G,E)\right) \geq \beta(F,E) \wedge \beta(G,E)$, $\forall (F,E),(G,E) \in SS(X,E)$,
then

$$\tau_\beta(F,E) = \underset{\underset{j \in J}{\cup}(G_j,E)=(F,E)}{\vee}\ \underset{j \in J}{\wedge}\beta(G_j,E),$$

is a gradation of openness and β is a base of τ_β.

Proof. From the condition a), $\tau_\beta(\Phi) = \tau_\beta\left(\widetilde{X}\right) = 1$ hold. For $\forall (F,E),(G,E) \in SS(X,E)$,

$$\begin{aligned}
\tau_\beta(F,E) \wedge \tau_\beta(G,E) &= \left(\bigvee_{\underset{\alpha\in A}{\cup}(F_\alpha,E)=(F,E)} \bigwedge_{\alpha\in A} \beta(F_\alpha,E)\right) \wedge \left(\bigvee_{\underset{\beta\in B}{\cup}(G_\beta,E)=(G,E)} \bigwedge_{\beta\in B} \beta(G_\beta,E)\right) \\
&= \bigvee_{\underset{\alpha\in A}{\cup}(F_\alpha,E)=(F,E)} \bigvee_{\underset{\beta\in B}{\cup}(G_\beta,E)=(G,E)} \left(\left(\bigwedge_{\alpha\in A}\beta(F_\alpha,E)\right) \wedge \left(\bigwedge_{\beta\in B}\beta(G_\beta,E)\right)\right) \\
&\le \bigvee_{\underset{\alpha\in A,\beta\in B}{\cup}\left((F_\alpha,E)\widetilde{\cap}(G_\beta,E)\right)=(F,E)\widetilde{\cap}(G,E)} \left(\bigwedge_{\alpha\in A,\beta\in B}\beta\left((F_\alpha,E)\cap(G_\beta,E)\right)\right) \\
&\le \bigvee_{\underset{\gamma\in C}{\cup}(H_\gamma,E)=(F,E)\widetilde{\cap}(G,E)} \bigwedge_{\gamma\in C}\beta(H_\gamma,E) \\
&= \tau_\beta\left((F,E)\,\widetilde{\cap}\,(G,E)\right)
\end{aligned}$$

is obtained. Now, let $\{(F_\lambda,E) : \lambda \in K\}$ be a family of soft sets. We consider a family

$$B_\lambda = \left\{ \{(G_{\delta_\lambda},E) : \delta_\lambda \in K_\lambda\} : \bigcup_{\delta_\lambda\in K_\lambda} (G_{\delta_\lambda},E) = (F_\lambda,E) \right\}.$$

Then

$$(F,E) = \bigcup_{\lambda\in K}(F_\lambda,E) = \bigcup_{\lambda\in K}\bigcup_{\delta_\lambda\in K_\lambda}(G_{\delta_\lambda},E).$$

For arbitrary $\rho \in \prod\limits_{\lambda\in K} B_\lambda$, since

$$\bigcup_{\lambda\in K}\bigcup_{\left(G_{\delta_\lambda},E\right)\in\rho(\lambda)}(G_{\delta_\lambda},E) = \bigcup_{\lambda\in K}(F_\lambda,E),$$

$$\begin{aligned}
\tau_\beta(F,E) &= \bigvee_{\underset{\delta\in K}{\cup}(G_\delta,E)=(F,E)} \bigwedge_{\delta\in K}\beta(G_\delta,E) \\
&\ge \bigvee_{\rho\in\prod\limits_{\lambda\in K}B_\lambda} \bigwedge_{\lambda\in K} \bigwedge_{\left(G_{\delta_\lambda},E\right)\in\rho(\lambda)} \beta(G_{\delta_\lambda},E) \\
&= \bigwedge_{\lambda\in K} \bigwedge_{\left\{\left(G_{\delta_\lambda},E\right):\delta_\lambda\in K_\lambda\right\}} \bigwedge_{\delta_\lambda\in K_\lambda} \beta(G_{\delta_\lambda},E) \\
&= \bigwedge_{\lambda\in K}\tau(F_\lambda,E),
\end{aligned}$$

is obtained. Thus τ_β is a gradation of openness. It is clear that β is a base of τ_β.

Theorem 3.5 *Let (X,E,τ) be a FTS and $Y \subset X$. Define mapping $\tau_Y : SS(Y,E) \to [0,1]$ by the rule*

$$\tau_Y(F,E) = \vee\left\{\tau(G,E) : (F,E) = (G,E)\,\widetilde{\cap}\widetilde{Y},\ (G,E)\in SS(X,E)\right\}.$$

Then τ_Y is a gradation of openness on Y and

$$\tau_Y\left((G,E)\,\widetilde{\cap}\widetilde{Y}\right) \ge \tau(G,E).$$

Proof. For each $(G,E) \in SS(X,E)$ with $(F,E) = (G,E)\,\widetilde{\cap}\widetilde{Y}$,

It is clear that $\tau_Y(\Phi) = \tau_Y\left(\widetilde{Y}\right) = 1$. Now,

$$\tau_Y\left((F_1,E)\,\widetilde{\cap}(F_2,E)\right) = \vee\left\{\tau(G,E) : (G,E)\,\widetilde{\cap}\widetilde{Y} = (F_1,E)\,\widetilde{\cap}(F_2,E)\right\}$$
$$\geq \vee\left\{\tau\left((G_1,E)\,\widetilde{\cap}(G_2,E)\right) : (G_1,E)\,\widetilde{\cap}\widetilde{Y} = (F_1,E),(G_2,E)\,\widetilde{\cap}\widetilde{Y} = (F_2,E)\right\}$$
$$\geq \left(\vee\left\{\tau(G_1,E) : (G_1,E)\,\widetilde{\cap}\widetilde{Y} = (F_1,E)\right\}\right) \wedge \left(\vee\left\{\tau(G_2,E) : (G_2,E)\,\widetilde{\cap}\widetilde{Y} = (F_2,E)\right\}\right)$$
$$= \tau_Y(F_1,E) \wedge \tau_Y(F_2,E),$$

is hold.

$$\begin{aligned}\tau_Y\left(\underset{i\in\Delta}{\cup}(F_i,E)\right) &= \vee\left\{\tau(G,E) : (G,E)\,\widetilde{\cap}\widetilde{Y} = \underset{i\in\Delta}{\cup}(F_i,E)\right\}\\ &\geq \vee\left\{\tau\left(\underset{i\in\Delta}{\cup}(G_i,E)\right) : (G_i,E)\widetilde{\cap}\widetilde{Y} = (F_i,E)\right\}\\ &\geq \vee\left\{\underset{i\in\Delta}{\wedge}\tau(G_i,E) : (G_i,E)\widetilde{\cap}\widetilde{Y} = (F_i,E)\right\}\\ &= \underset{i\in\Delta}{\wedge}\left(\vee\left\{\tau(G_i,E) : (G_i,E)\widetilde{\cap}\widetilde{Y} = (F_i,E)\right\}\right)\\ &= \underset{i\in\Delta}{\wedge}\tau_Y(F_i,E),\end{aligned}$$

Hence τ_Y is a gradation of openness on Y and $\tau_Y\left((G,E)\,\widetilde{\cap}\widetilde{Y}\right) \geq \tau(G,E)$ holds.

Definition 3.4 *Let (X,E,τ) and (Y,E',γ) be two $FTSs$ and $(f,\varphi) : (X,E,\tau) \to (Y,E',\gamma)$ be a mapping. Then (f,φ) is called a continuous mapping at the soft point $x_e \in (X,E)$ if for each arbitrary soft set $(f,\varphi)(x_e) = (f(x))_{\varphi(e)} \in (G,E') \in SS(Y,E')$, there exists $(F,E) \in SS(X,E)$ such that $x_e \in (F,E)$,*

$$\tau(F,E) \geq \gamma\left(G,E'\right) \text{ and } (f,\varphi)(F,E) \subset \left(G,E'\right).$$

If (f,φ) is a continuous mapping for each soft point, then (f,φ) is a continuous mapping.

Theorem 3.6 *Let (X,E,τ) and (Y,E',γ) be two $FTSs$ and $(f,\varphi) : (X,E,\tau) \to (Y,E',\gamma)$ be a mapping. Then (f,φ) is a continuous mapping if and only if*

$$\tau\left((f,\varphi)^{-1}\left(G,E'\right)\right) \geq \gamma\left(G,E'\right)$$

is satisfied, $\forall\,(G,E') \in SS(Y,E')$.

Proof. Let (f,φ) be a continuous mapping and $(G,E') \in SS(Y,E')$ be an arbitrary soft set. Suppose $x_e \in (f,\varphi)^{-1}(G,E')$ be an arbitrary soft point. Since (f,φ) is a continuous mapping, there exists $(F,E)_{x_e} \in SS(X,E)$ such that $x_e \in (F,E)$,

$$\tau(F,E)_{x_e} \geq \gamma\left(G,E'\right) \text{ and } (f,\varphi)(F,E)_{x_e} \subset \left(G,E'\right).$$

Then

$$(f,\varphi)^{-1}\left(G,E'\right) = \underset{x_e\in(f,\varphi)^{-1}(G,E')}{\cup} x_e \subset \underset{x_e\in(f,\varphi)^{-1}(G,E')}{\cup}(F,E)_{x_e} \subset (f,\varphi)^{-1}\left(G,E'\right).$$

We have

$$\tau\left((f,\varphi)^{-1}\left(G,E'\right)\right)=\tau\left(\underset{x_e}{\cup}(F,E)_{x_e}\right)\geq\wedge\tau\left(F,E\right)_{x_e}\geq\gamma\left(G,E'\right).$$

Conversely, let $x_e\in SS(X,E)$ be an arbitrary soft point and $(f,\varphi)(x_e)\in(G,E')$. From the condition of theorem, $x_e\in(f,\varphi)^{-1}(G,E')$,

$$\tau\left((f,\varphi)^{-1}\left(G,E'\right)\right)\geq\gamma\left(G,E'\right),$$

and $(f,\varphi)\left((f,\varphi)^{-1}(G,E')\right)\subset(G,E')$ are satisfied. Thus (f,φ) is a continuous mapping.

Theorem 3.7 *Let (X,E,τ) and (Y,E',γ) be two $FTSs$ and $(f,\varphi):(X,E,\tau)\to(Y,E',\gamma)$ be a mapping. Then (f,φ) is a continuous mapping if and only if $(f_r,\varphi_r):(X,E,\tau_r)\to(Y,E',\gamma_r)$ is a continuous mapping on soft bitopological space for each $r\in(0,1]$.*

Proof. Suppose (f,φ) is a continuous mapping and $(G,E')\in\gamma_r$. Then $\gamma(G,E')\geq r$. Since

$$\tau\left((f,\varphi)^{-1}\left(G,E'\right)\right)\geq\gamma\left(G,E'\right)\geq r,$$

$(f,\varphi)^{-1}(G,E')\in\tau_r$.

Conversely, suppose (f_r,φ_r) is a continuous mapping for $\forall r\in(0,1]$. If $\gamma(G,E')=r$ for $\forall(G,E')\in SS(Y,E')$, then $(G,E')\in\gamma_r$. Since (f_r,φ_r) is a continuous mapping, $(f_r,\varphi_r)^{-1}(G,E')\in\tau_r$. Then

$$\tau\left((f,\varphi)^{-1}\left(G,E'\right)\right)\geq r=\gamma\left(G,E'\right).$$

Thus $(f,\varphi):(X,E,\tau)\to(Y,E',\gamma)$ is a continuous mapping.

Theorem 3.8 *Let (X,E,τ) and (Y,E',γ) be two $FTSs$ and β be a base of γ on Y. Then $(f,\varphi):(X,E,\tau)\to(Y,E',\gamma)$ is a continuous mapping if and only if $\beta(G,E')\leq\tau\left((f,\varphi)^{-1}(G,E')\right)$ for $\forall(G,E')\in SS(Y,E')$.*

Proof. Let $(f,\varphi):(X,E,\tau)\to(Y,E',\gamma)$ be a continuous mapping and $(G,E')\in SS(Y,E')$. Then $\gamma(G,E')\geq\beta(G,E')$. So

$$\tau\left((f,\varphi)^{-1}\left(G,E'\right)\right)\geq\gamma\left(G,E'\right)\geq\beta\left(G,E'\right),$$

is obtained.

Conversely, let $\beta(G,E')\leq\tau\left((f,\varphi)^{-1}(G,E')\right)$, for $\forall(G,E')\in SS(Y,E')$. Let $(G,E')=\underset{i\in I}{\cup}(G_i,E')$. We have

$$\begin{aligned}\tau\left((f,\varphi)^{-1}\left(G,E'\right)\right)&=\tau\left((f,\varphi)^{-1}\left(\underset{i\in I}{\cup}\left(G_i,E'\right)\right)\right)\\&=\tau\left(\underset{i\in I}{\cup}(f,\varphi)^{-1}\left(G_i,E'\right)\right)\\&\geq\underset{i\in I}{\wedge}\tau\left((f,\varphi)^{-1}\left(G_i,E'\right)\right)\\&\geq\underset{i\in I}{\wedge}\beta\left(G_i,E'\right).\end{aligned}$$

Since this equality is satisfied for arbitrary $(G,E') = \underset{i\in I}{\cup}(G_i,E')$,

$$\tau\left((f,\varphi)^{-1}(G,E')\right) \geq \bigvee_{(G,E')=\underset{i\in I}{\cup}(G_i,E')} \bigwedge_{i\in I} \beta(G_i,E') = \gamma(G,E').$$

is obtained.

Theorem 3.9 *Let (X,E,τ) and (Y,E',γ) be two $FTSs$ and δ be a subbase of γ. If*

$$\delta(G,E') \leq \tau\left((f,\varphi)^{-1}(G,E')\right), \forall (G,E') \in SS(Y,E')$$

is satisfied, then $(f,\varphi) : (X,E,\tau) \to (Y,E',\gamma)$ is a continuous mapping.

Proof. For $\forall (G,E') \in SS(Y,E')$,

$$\begin{aligned}
\delta(G,E') &= \bigvee_{\underset{\lambda\in K}{\cup}(G_\lambda,E')=(G,E')} \bigwedge_{\lambda\in K} \bigvee_{\underset{\mu\in K_\lambda}{\cap}(F_\lambda,E')=(G_\lambda,E')} \bigwedge_{\mu\in K_\lambda} \gamma((F_\mu,E')) \\
&\leq \bigvee_{\underset{\lambda\in K}{\cup}(G_\lambda,E')=(G,E')} \bigwedge_{\lambda\in K} \bigvee_{\underset{\mu\in K_\lambda}{\cap}(F_\lambda,E')=(G_\lambda,E')} \bigwedge_{\mu\in K_\lambda} \tau\left((f,\varphi)^{-1}(F_\mu,E')\right) \\
&\leq \bigvee_{\underset{\lambda\in K}{\cup}(G_\lambda,E')=(G,E')} \bigwedge_{\lambda\in K} \tau\left((f,\varphi)^{-1}(G_\lambda,E')\right) \\
&\leq \bigvee_{\underset{\lambda\in K}{\cup}(G_\lambda,E')=(G,E')} \tau\left((f,\varphi)^{-1}\left(\underset{\lambda\in K}{\cup}(G_\lambda,E')\right)\right) \\
&= \tau\left((f,\varphi)^{-1}(G,E')\right)
\end{aligned}$$

is obtained.

Definition 3.5 *Let (X,E,τ) and (Y,E',γ) be two $FTSs$ and (f,φ) be a mapping from (X,E,τ) into (Y,E',γ). The mapping (f,φ) is called an open mapping if it satisfies the following condition:*

$$\tau(F,E) \leq \gamma((f,\varphi)(F,E)),\ \forall (F,E) \in SS(X,E).$$

Theorem 3.10 *Let (X,E,τ) and (Y,E',γ) be two $FTSs$ and $(f,\varphi) : (X,E,\tau) \to (Y,E',\gamma)$ be a mapping and β be a base of τ. If*

$$\beta(F,E) \leq \gamma((f,\varphi)(F,E)), \forall (F,E) \in SS(X,E)$$

is satisfied, then (f,φ) is an open mapping.

Proof. For $\forall (F,E) \in SS(X,E)$

$$\begin{aligned}
\tau(F,E) &= \bigvee_{\underset{i\in I}{\cup}(F_i,E)=(F,E)} \bigwedge_{i\in I} \beta((F_i,E)) \\
&\leq \bigvee_{\underset{i\in I}{\cup}(F_i,E)=(F,E)} \bigwedge_{i\in I} \gamma((f,\varphi)(F_i,E)) \\
&\leq \bigvee_{\underset{i\in I}{\cup}(F_i,E)=(F,E)} \gamma\left((f,\varphi)\left(\underset{i\in I}{\cup}(F_i,E)\right)\right) \\
&= \gamma((f,\varphi)(F,E))
\end{aligned}$$

is satisfied.

Now by using the mapping $(f,\varphi): SS(X,E) \to (Y,E',\gamma)$ and gradation of openness γ, we define gradation of openness on $SS(X,E)$ such that (f,φ) is a continuous mapping.

Theorem 3.11 *Let (Y,E',γ) be a FTS and $(f,\varphi): SS(X,E) \to (Y,E',\gamma)$ be a mapping of soft sets. Then define $\tau : SS(X,E) \to [0,1]$ by:*

$$\tau(F,E) = \underset{f^{-1}(G,E')=(F,E)}{\vee} \gamma(G,E').$$

Then τ is a gradation of openness on X and (f,φ) is a continuous mapping.

Proof. It is clear that $\tau(\Phi) = \tau\left(\widetilde{X}\right) = 1$. Now,

$$\begin{aligned}
\tau\left((F_1,E)\,\widetilde{\cap}(F_2,E)\right) &= \vee\left\{\gamma(G,E') : (f,\varphi)^{-1}(G,E') = (F_1,E)\,\widetilde{\cap}(F_2,E)\right\} \\
&\geq \vee\left\{\gamma\left((G_1,E')\,\widetilde{\cap}(G_2,E')\right) : (f,\varphi)^{-1}\left((G_1,E')\,\widetilde{\cap}(G_2,E')\right) = (F_1,E)\,\widetilde{\cap}(F_2,E)\right\} \\
&\geq \left(\vee\left\{\gamma(G_1,E') : (f,\varphi)^{-1}(G_1,E') = (F_1,E)\right\}\right)\wedge\left(\vee\left\{\gamma(G_2,E') : (f,\varphi)^{-1}(G_2,E') = (F_2,E)\right\}\right) \\
&= \tau(F_1,E)\wedge\tau(F_2,E)
\end{aligned}$$

is hold. Furthermore,

$$\begin{aligned}
\tau\left(\underset{i\in\Delta}{\cup}(F_i,E)\right) &= \vee\left\{\gamma(G,E') : (f,\varphi)^{-1}(G,E') = \underset{i\in\Delta}{\cup}(F_i,E)\right\} \\
&\geq \vee\left\{\gamma\left(\underset{i\in\Delta}{\cup}(G_i,E')\right) : (f,\varphi)^{-1}\left(\underset{i\in\Delta}{\cup}(G_i,E')\right) = \underset{i\in\Delta}{\cup}(F_i,E)\right\} \\
&\geq \vee\left\{\underset{i\in\Delta}{\wedge}\gamma(G_i,E') : (f,\varphi)^{-1}(G_i,E') = (F_i,E)\right\} \\
&= \underset{i\in\Delta}{\wedge}\left(\vee\left\{\gamma(G_i,E') : (f,\varphi)^{-1}(G_i,E') = (F_i,E)\right\}\right) \\
&= \underset{i\in\Delta}{\wedge}\tau(F_i,E).
\end{aligned}$$

It implies that τ is a gradation of openness on X, and (f,φ) is a continuous mapping.

Theorem 3.12 *Let (X,E,τ) be a FTS and $(f,\varphi): (X,E,\tau) \to SS(Y,E')$ be a mapping of soft sets. Then define $\gamma : SS(Y,E') \to [0,1]$ by:*

$$\gamma(G,E') = \tau\left((f,\varphi)^{-1}(G,E')\right),$$

is a gradation of openness on Y and (f,φ) is a continuous mapping.

Proof. It is clear that $\gamma(\Phi) = \gamma\left(\widetilde{Y}\right) = 1$. Now,

$$\begin{aligned}
\gamma\left((G_1,E')\,\widetilde{\cap}(G_2,E')\right) &= \tau\left((f,\varphi)^{-1}\left((G_1,E')\,\widetilde{\cap}(G_2,E')\right)\right) \\
&= \tau\left((f,\varphi)^{-1}(G_1,E')\,\widetilde{\cap}\,(f,\varphi)^{-1}(G_2,E')\right) \\
&\geq \tau\left((f,\varphi)^{-1}(G_1,E')\right)\wedge\tau\left((f,\varphi)^{-1}(G_2,E')\right) \\
&= \gamma(G_1,E')\wedge\gamma(G_2,E'),
\end{aligned}$$

is obtained. Moreover,

$$\begin{aligned}
\gamma\left(\underset{i\in\Delta}{\cup}(G_i,E')\right) &= \tau\left((f,\varphi)^{-1}\left(\underset{i\in\Delta}{\cup}(G_i,E')\right)\right) \\
&= \tau\left(\underset{i\in\Delta}{\cup}(f,\varphi)^{-1}(G_i,E')\right) \\
&\geq \underset{i\in\Delta}{\wedge}\tau\left((f,\varphi)^{-1}(G_i,E')\right) = \underset{i\in\Delta}{\wedge}\gamma(G_i,E').
\end{aligned}$$

Thus γ is a gradation of openness on Y. From the definition of topology, (f,φ) is a continuous mapping.

Now we define the concept of quotient space of $FTSs$.

Let $\{(X_\lambda, E_\lambda, \tau_\lambda)\}_{\lambda\in\wedge}$ be a family of fuzzy topological spaces, different $X_\lambda \cap X_{\lambda'} = \varnothing$ and $E_\lambda \cap E_{\lambda'} = \varnothing$, $\forall \lambda \neq \lambda'$. Let $\widetilde{X}$ be union of all soft points which belong to this space and $E = \underset{\lambda\in\wedge}{\cup} E_\lambda$. Then $\left(\widetilde{X}, E\right)$ is a family of soft sets on $X = \underset{\lambda\in\wedge}{\cup} X_\lambda$ with parameters E. For soft point $x_e \in \left(\widetilde{X}, E\right)$, if $x \in X_\lambda$ then $e \in E_\lambda$. If $e \in E_\lambda$, then $x \in X_\lambda$ is satisfied. For arbitrary $(F, E) \in \left(\widetilde{X}, E\right)$, $(F, E)_\lambda = \{F(e) \cap X_\lambda\}_{e\in E}$.

Theorem 3.13 *Let $\{(X_\lambda, E_\lambda, \tau_\lambda)\}_{\lambda\in\wedge}$ be a family of $FTSs$, different $X_\lambda' s$ be disjoint. Then τ which is defined as follows*

$$\tau(F, E) = \underset{\lambda\in\wedge}{\wedge} \tau_\lambda\left((F, E)_\lambda\right), \forall (F, E) \in \left(\widetilde{X}, E\right)$$

is gradation of openness on X.

Proof. First let $(F_1, E), (F_2, E) \in \left(\widetilde{X}, E\right)$. Then

$$\begin{aligned}
\tau\left((F_1, E)\,\widetilde{\cap}(F_2, E)\right) &= \underset{\lambda\in\wedge}{\wedge} \tau_\lambda\left(\left((F_1, E)\,\widetilde{\cap}(F_2, E)\right)_\lambda\right) \\
&= \underset{\lambda\in\wedge}{\wedge} \tau_\lambda\left((F_1, E)_\lambda\,\widetilde{\cap}(F_2, E)_\lambda\right) \\
&\geq \underset{\lambda\in\wedge}{\wedge} \left(\tau_\lambda\left((F_1, E)_\lambda\right) \wedge \tau_\lambda\left((F_2, E)_\lambda\right)\right) \\
&= \left(\underset{\lambda\in\wedge}{\wedge} \tau_\lambda\left((F_1, E)_\lambda\right)\right) \wedge \left(\underset{\lambda\in\wedge}{\wedge} \tau_\lambda\left((F_2, E)_\lambda\right)\right) \\
&= \tau(F_1, E) \wedge \tau(F_2, E)
\end{aligned}$$

is satisfied.

Secondly, for $\{(F_i, E_i)\}_{i\in I}$ be a family of soft sets,

$$\begin{aligned}
\tau\left(\underset{i\in I}{\cup}(F_i, E_i)\right) &= \underset{\lambda\in\wedge}{\wedge} \tau_\lambda\left(\left(\underset{i\in I}{\cup}(F_i, E_i)\right)_\lambda\right) \\
&= \underset{\lambda\in\wedge}{\wedge} \tau_\lambda\left(\underset{i\in I}{\cup}(F_i, E_i)_\lambda\right) \\
&\geq \underset{\lambda\in\wedge}{\wedge}\ \underset{i\in I}{\wedge} \tau_\lambda\left((F_i, E_i)_\lambda\right) \\
&= \underset{i\in I}{\wedge}\left(\underset{\lambda\in\wedge}{\wedge} \tau_\lambda\left((F_i, E_i)_\lambda\right)\right) = \underset{i\in I}{\wedge} \tau\left((F_i, E_i)\right),
\end{aligned}$$

is obtained. Thus (X, E, τ) is a FTS.

Definition 3.6 *The fuzzy topological space (X, E, τ) is called the direct sum of $\{(X_\lambda, E_\lambda, \tau_\lambda)\}_{\lambda\in\wedge}$, denoted by*

$$(X, E, \tau) = \underset{\lambda\in\wedge}{\oplus} (X_\lambda, E_\lambda, \tau_\lambda).$$

It is clear that since $i_\lambda : X_\lambda \to X = \underset{\lambda\in\wedge}{\cup} X_\lambda$ and $j_\lambda : E_\lambda \to E = \underset{\lambda\in\wedge}{\cup} E_\lambda$ are embedding mappings for all $\lambda \in \wedge$, the mapping

$$(i_\lambda, j_\lambda) : (X_\lambda, E_\lambda, \tau_\lambda) \to (X, E, \tau)$$

is a continuous mapping.

Theorem 3.14 *Let $\{(X_\lambda, E_\lambda, \tau_\lambda)\}_{\lambda\in\wedge}$ be a family of $FTSs$, $X = \prod\limits_{\lambda\in\wedge} X_\lambda$ be a set, $E = \prod\limits_{\lambda\in\wedge} E_\lambda$ be a parameter set and for each $\lambda \in \Lambda$, $p_\lambda : X \to X_\lambda$ and $q_\lambda : E \to E_\lambda$ be two projections maps. Define $\beta : SS(Y, E) \to [0, 1]$ by:*

$$\beta\left(G, E\right) = \vee\left\{\overset{n}{\underset{j=1}{\wedge}} \tau_{\alpha_j}\left(F_{\alpha_j}, E_{\alpha_j}\right) : (F, E) = \overset{n}{\underset{j=1}{\cap}} \left(p_{\alpha_j}, q_{\alpha_j}\right)^{-1}\left(F_{\alpha_j}, E_{\alpha_j}\right)\right\}.$$

Then β is a base on FTS and for each $\lambda \in \Lambda$, $(p_\lambda, q_\lambda) : (X, E, \tau_\beta) \to (X_\lambda, E_\lambda, \tau_\lambda)$ are continuous maps.

Proof. Now we check conditions of base for β.

$$\begin{aligned}\beta\left(\widetilde{X}\right) &= \vee\left\{\overset{n}{\underset{j=1}{\wedge}} \tau_{\alpha_j}\left(F_{\alpha_j}, E_{\alpha_j}\right) : \widetilde{X} = \overset{n}{\underset{j=1}{\cap}} \left(p_{\alpha_j}, q_{\alpha_j}\right)^{-1}\left(F_{\alpha_j}, E_{\alpha_j}\right)\right\}\\ &= \vee\left\{\overset{n}{\underset{j=1}{\wedge}} \tau_{\alpha_j}\left(X_{\alpha_j}, E_{\alpha_j}\right)\right\} = 1\end{aligned}$$

is holds. Similarly, $\beta\left(\Phi\right) = 1$ is obtained. Now,

$$\begin{aligned}\beta\left(F, E\right) \wedge \beta\left(G, E\right) &= \left(\bigvee_{\overset{n}{\underset{j=1}{\cap}}\left(p_{\alpha_j}, q_{\alpha_j}\right)^{-1}\left(F_{\alpha_j}, E_{\alpha_j}\right) = (F,E)} \overset{n}{\underset{j=1}{\wedge}} \tau_{\alpha_j}\left(F_{\alpha_j}, E_{\alpha_j}\right)\right) \wedge\\ &\quad \left(\bigvee_{\overset{k}{\underset{i=1}{\cap}}\left(p_{\gamma_i}, q_{\gamma_i}\right)^{-1}\left(G_{\gamma i}, E_{\gamma_i}\right) = (G,E)} \overset{k}{\underset{i=1}{\wedge}} \tau_{\gamma_i}\left(G_{\gamma_i}, E_{\gamma_i}\right)\right)\\ &= \bigvee_{\overset{n}{\underset{j=1}{\cap}}\left(p_{\alpha_j}, q_{\alpha_j}\right)^{-1}\left(F_{\alpha_j}, E_{\alpha_j}\right) = (F,E)} \bigvee_{\overset{k}{\underset{i=1}{\cap}}\left(p_{\gamma_i}, q_{\gamma_i}\right)^{-1}\left(G_{\gamma i}, E_{\gamma_i}\right) = (G,E)}\\ &\quad \left(\left(\overset{n}{\underset{j=1}{\wedge}} \tau_{\alpha_j}\left(F_{\alpha_j}, E_{\alpha_j}\right)\right) \wedge \left(\overset{k}{\underset{i=1}{\wedge}} \tau_{\gamma_i}\left(G_{\gamma_i}, E_{\gamma_i}\right)\right)\right)\\ &= \bigvee_{\cap\left(\left(\left(p_{\alpha_j}, q_{\alpha_j}\right)^{-1}\left(F_{\alpha_j}, E_{\alpha_j}\right)\right) \cap \left(\left(p_{\gamma_i}, q_{\gamma_i}\right)^{-1}\left(G_{\gamma i}, E_{\gamma_i}\right)\right)\right) = (F,E)\cap(G,E)}\\ &\quad \left(\left(\overset{n}{\underset{j=1}{\wedge}} \tau_{\alpha_j}\left(F_{\alpha_j}, E_{\alpha_j}\right)\right) \wedge \left(\overset{k}{\underset{i=1}{\wedge}} \tau_{\gamma_i}\left(G_{\gamma_i}, E_{\gamma_i}\right)\right)\right)\\ &\leq \bigvee_{\underset{\lambda}{\cap}\left(p_{\theta_\lambda}, q_{\theta_\lambda}\right)^{-1}\left(H_{\theta_\lambda}, E_{\theta_\lambda}\right) = (F,E)\cap(G,E)} \tau_{\theta_\lambda}\left(H_{\theta_\lambda}, E_{\theta_\lambda}\right)\\ &= \beta\left(\left(F_1, E\right) \widetilde{\cap} \left(F_2, E\right)\right).\end{aligned}$$

Thus β is satisfied conditions of base. Now we show that the projection mapping $(p_\lambda, q_\lambda) : (X, E, \tau_\beta) \to (X_\lambda, E_\lambda, \tau_\lambda)$ are continuous maps, for $\forall\lambda \in \Lambda$. Indeed for $\forall\left(F_\lambda, E_\lambda\right) \in SS\left(X_\lambda, E_\lambda\right)$,

$$\begin{aligned}\tau\left(\left(p_\lambda, q_\lambda\right)^{-1}\left(F_\lambda, E_\lambda\right)\right) &\geq \beta\left(\left(p_\lambda, q_\lambda\right)^{-1}\left(F_\lambda, E_\lambda\right)\right)\\ &= \vee\left\{\overset{n}{\underset{j=1}{\wedge}} \tau_{\alpha_j}\left(F_{\alpha_j}, E_{\alpha_j}\right) : \left(p_{\alpha_j}, q_{\alpha_j}\right)^{-1}\left(F_{\alpha_j}, E_{\alpha_j}\right) = \left(p_\lambda, q_\lambda\right)^{-1}\left(F_\lambda, E_\lambda\right)\right\}\\ &\geq \tau_\lambda\left(F_\lambda, E_\lambda\right)\end{aligned}$$

is obtained. Thus the proof is completed.

4 CONCLUSION

Soft topological spaces are an important generalization of topological spaces. In this paper, we give the definition of gradation of openness τ which is a mapping from $SS(X, E)$ to $[0, 1]$ which satisfies some definite conditions. Then we show that fuzzy topological space gives a parameterized family of soft bitopologies on X. Later we introduce the concepts of base and subbase in fuzzy topological space of soft sets and use them to discuss continuous mapping and open mapping. We hope that the results of this study may help in the investigation of fuzzy topological spaces on soft sets.

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