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Absence of global solutions of a semilinear biharmonic equation with a singular potential in exterior domains

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Abstract. *In the domain* $\{x \in \mathbb{R}^n : |x| > R\} \times \{y \in \mathbb{R}^m : |y| > R\}$ *the problem of the absence of global solutions of a semilinear biharmonic equation* $\Delta_x\left(|x|^{\alpha}\Delta_x u\right) + \Delta_y^2 u - \frac{C_1}{|x|^{4-\alpha}}u - \frac{C_2}{|y|^4}u - |x|^{\sigma_1}|y|^{\sigma_2}|u|^q = 0$ *with are conditions* $u \geq 0, \Delta u \leq 0$ *on* $\{|x| = R\} \times \{|y| > R\} \cup \{|x| > R\} \times \{|y| = R\}$ *is investigated, where* $q > 1, \alpha < 4, n > 4 - \alpha, m > 4, 0 \leq C_1 < \left(\frac{(n-\alpha)(n+\alpha-4)}{4}\right)^2, 0 \leq C_2 < \left(\frac{m(m-4)}{4}\right)^2$. *A sufficient condition for the absence of global solutions is obtained. The proof is based on the test function method.*

Keywords. Semilinear biharmonic equation, global solution, singular potential, critical exponent, test function method.

Mathematics Subject Classification (2010): 35A01, 35B33, 35J61, 35J30

1 Introduction

We induduce the following designations:

$$
\begin{aligned}
&x = (x_1, ..., x_n) \in \mathbb{R}^n,\ \ y = (y_1, ..., y_m) \in \mathbb{R}^m, |x| = \sqrt{\sum_{i=1}^{n} x_i^2}, |y| = \sqrt{\sum_{i=1}^{m} y_i^2},\\
&B_x(r) = \{x \in \mathbb{R}^n : |x| < r\}, B_y(r) = \{y \in \mathbb{R}^m : |y| < r\},\\
&S_x(r) = \{x \in \mathbb{R}^n : |x| = r\}, S_y(r) = \{y \in \mathbb{R}^m : |y| = r\},\\
&B(r) = B_x(r) \times B_y(r), B_x(r_1, r_2) = \{x \in \mathbb{R}^n : r_1 < |x| < r_2\},\\
&B_y(r_1, r_2) = \{y \in \mathbb{R}^m : r_1 < |y| < r_2\},\\
&B(r_1, r_2) = B_x(r_1, r_2) \times B_y(r_1, r_2), B_x^{'}(R) = \mathbb{R}^n \backslash B_x(R), B_y^{'}(R) = \mathbb{R}^m \backslash B_y(R),\\
&B'(R) = B_x^{'}(R) \times B_y^{'}(R),\ \partial B^{'}(R) = S_x(R) \times B_y^{'}(R) \cup B_x^{'}(R) \times S_y(R),
\end{aligned}
$$

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$$B_x^{'}(1) = B_x^{'}, B_y^{'}(1) = B_y^{'}, B^{'}(1) = B'.$$

In $B'(R)$ we consider the equation

$$\Delta_x \left(|x|^\alpha \Delta_x u\right) + \Delta_y^2 u - \frac{C_1}{|x|^{4-\alpha}} u - \frac{C_2}{|y|^4} u - |x|^{\sigma_1} |y|^{\sigma_2} |u|^q = 0, \tag{1.1}$$

where

$$\alpha < 4, \sigma_1, \sigma_2 \in R, q > 1, 0 \le C_1 < \left(\frac{(n-\alpha)(n+\alpha-4)}{4}\right)^2,$$

$$0 \le C_2 < \left(\frac{m-(m-4)}{4}\right)^2, \Delta_x = \sum_{i=1}^{n} \frac{\partial^2}{\partial x_i^2}, \Delta_y^2 = \Delta_y(\Delta_y), \Delta_y = \sum_{i=1}^{m} \frac{\partial^2}{\partial y_i^2}.$$

We will consider the existence of the solution of an equation satisfying the following conditions:

$$u \ge 0, \Delta u \le 0 \text{ on } \partial B'(R). \tag{1.2}$$

We will understand the solution of problem (1.1),(1.2) in the classic sense, namely, the function $u(x,y) \in C^4(B_R^{'}) \cup C^3(B_R^{'} \cup \partial B_R^{'})$ satisfying equation (1.1) at each point of $B'(R)$ and condition (1.2) on $\partial B'(R)$ will be called global solutions of problem (1.1),(1.2).

The problem of non- existence of global solutions for a different class of differential equations and inequalities play an important role in theory and applications; therefore, constantly draw attention of mathematicians and there is a large number of works devoted to them. A survey of such results was given in the monograph [15]. It is well known such statements are a direct analogs of the Liouvilles theorem from the theory of functions of a complex variable.

The issues considered in this paper were earlier studied in the works of Mitidieri and Pokhozhaev [15], Gidas and Spruck [9], Brezis, Dupaigne and Tesei [6], Bidaut-Veron and Pohozaev [5], Serrin and Zou [18], Serrin [19], Konkov [10,11,12], Bagirov [1,2], Bagirov, Guliyev [4], Mamedov [14]. Similar issues for weakly nonlinear equations with a biharmonic operator were considered by various authors as Laptev [13], Volodin [20,21], Xu [22], Ghergu and Taliaferro [8], Carriao, Demarque and Miyagaki [7], Yao,Wang, Shen [23], Bagirov [3].

In the paper [21] for $\alpha = 0, C_1 = C_2 = 0$ is considered in $B'(R)$ with different boundary conditions on S_R and critical exponents of the absence of the solutions of the considered problems were found. In the paper [3] for $\alpha = 0$ the problem (1.1), (1,2) is considered in $B'(R)$ and critical exponent of the absence global solutions are also found.

In this paper the problem (1.1), (1,2) is consider for $\alpha < 4, n > 4-\alpha, m > 4$, $0 \le C_1 < \left(\frac{(n-\alpha)(n+\alpha-4)}{4}\right)$, $0 \le C_2 < \left(\frac{(m(m-4)}{4}\right)^2$ and exact estimates of the absence global solutions are obtained. The technique of the test functions developed by Mitidieri and Pohozhayev in the paper [15,16,17] is used.

2 Main result and its proof

We introduce the following designation:

$$D_1 = \sqrt{\left(\frac{(n-2)(\alpha-2)}{2}\right)^2 + C_1}, \; D_2 = \sqrt{(m-2)^2 + C_2}$$

$$\alpha_\pm = \sqrt{\left(\frac{n-2}{2}\right)^2 + \left(\frac{\alpha-2}{2}\right)^2 \mp D_1} \quad \text{for } 2 \le \alpha < 4,$$

$$\alpha_{\pm}=\sqrt{\left(\frac{n-2}{2}\right)^2+\left(\frac{\alpha-2}{2}\right)^2\pm D_1}\quad \text{for }\ \alpha<2,$$

$$\beta_{\pm}=\sqrt{\left(\frac{m-2}{2}\right)^2+1\pm\sqrt{D_2}},\ \ \gamma=\frac{\frac{2\sqrt{D_1}}{|\alpha-2|}-\alpha_+}{\alpha_-},$$

$$a=\frac{m+4}{2}+\beta_- -\sigma_2\frac{1}{q-1}=\frac{m+4}{2}+\beta_- -\sigma_2(q'-1),$$

$$b=\frac{n+4-\alpha}{2}+\alpha_- -\sigma_1(q'-1),$$

where $\frac{1}{q}+\frac{1}{q'}=1$.

Let us consider the following functions

$$\xi(x)=\xi(|x|)=\frac{1}{2}(1+\gamma)|x|^{-\frac{n+\alpha-4}{2}+\alpha_-}+\frac{1}{2}(1-\gamma)|x|^{-\frac{n+\alpha-4}{2}-\alpha_-}$$
$$-|x|^{-\frac{n+\alpha-4}{2}-\alpha_+}\ \text{for }\alpha\neq 2,$$

$$\xi(x)=\frac{\alpha_-}{\alpha_+}\left(|x|^{-\frac{n-2}{2}+\alpha_-}-|x|^{-\frac{n-2}{2}-\alpha_-}\right)+|x|^{-\frac{n-2}{2}+\alpha_+}-|x|^{-\frac{n-2}{2}-\alpha_+}\ \text{for }\alpha=2,$$

$$\eta(y)=\eta(|y|)=\frac{1}{2}\left(1+\frac{\sqrt{D_2}-\beta_+}{\beta_-}\right)|y|^{-\frac{m-4}{2}+\beta_-}$$
$$+\frac{1}{2}\left(1-\frac{\sqrt{D_2}-\beta_+}{\beta_-}\right)|y|^{-\frac{m-4}{2}-\beta_-}-|y|^{-\frac{m-4}{2}-\beta_+}.$$

We can show that $\xi(x)$ is the solution of the equation

$$\Delta_x(|x|^{\alpha}\Delta_x u)-\frac{C_1}{|x|^{4-\alpha}}\ u=0, \tag{2.1}$$

$\eta(y)$ is the solution of the equation

$$\Delta_y^2 u-\frac{C_2}{|y|^4}u=0, \tag{2.2}$$

and

$$\xi(x)|_{|x|=1}=0,\ \ \left.\frac{\partial\xi}{\partial r}\right|_{|x|=1}>0,\ \ \Delta_x\xi|_{|x|=1}=0\ ,\ \left.\frac{\Delta_x\xi}{\partial r}\right|_{|x|=1}\leq 0,$$

$$\eta(x)|_{|y|=1}=0,\ \ \left.\frac{\partial\eta}{\partial r}\right|_{|y|=1}>0,\ \ \Delta_y\eta|_{|y|=1}=0\ ,\ \left.\frac{\Delta_y\eta}{\partial r}\right|_{|y|=1}\leq 0.$$

We consider the following functions $\varphi(x),\psi(y)$ such that $\varphi(x)\in C^{\infty}(\mathbb{R}^n),\varphi(x)=1$ for $|x|\leq\rho,\varphi(x)=0$ for $|x|\geq 2\rho$, $\psi(y)\in C^{\infty}(\mathbb{R}^m)$, $\psi(y)=1$ for $|y|\leq\rho^{\varepsilon},\psi(y)=0$ for $|y|\geq 2\rho^{\varepsilon}$.

The following theorem is the main result of this paper.

Theorem 2.1 *Let* $\alpha < 4, n > 4-\alpha, m > 4, 0 \le C_1 < \left(\frac{(n-\alpha)(n+\alpha-4)}{4}\right)^2, 0 \le C_2 < \left(\frac{m(m-4)}{4}\right)^2$ *and*

a) $\sigma_1 > 0, \sigma_2 > 0, 1 < q \le \min\left\{1 + \frac{\sigma_1}{\frac{n-\alpha+4}{2}+\alpha_-}, 1 + \frac{\sigma_2}{\frac{m+4}{2}+\beta_-}\right\}$,

b) $\sigma_1 > 0, \sigma_2 > -4, q > \max\left\{1, 1 + \frac{\sigma_2}{\frac{m+4}{2}+\beta_-}\right\}$ *and* $q < 1 + \frac{\sigma_1}{\frac{n-\alpha+4}{2}+\alpha_-}$,

$$q \le 1 + \frac{\sigma_2 + 4}{\frac{m+4}{2} + \beta_- - 4} \text{ or } q = 1 + \frac{\sigma_1}{\frac{n-\alpha+4}{2} + \alpha_-}, \ \ q < 1 + \frac{\sigma_2 + 4}{\frac{m+4}{2} + \beta_- - 4},$$

c) $\sigma_1 > \alpha - 4, \sigma_2 > 0, \ \ q > \max\left\{1, 1 + \frac{\sigma_1}{\frac{n-\alpha+4}{2}+\alpha_-}\right\}$ *and* $q < 1 + \frac{\sigma_2}{\frac{m+4}{2}+\beta_-}$,

$$q \le 1 + \frac{\sigma_1 + 4 - \alpha}{\frac{n+\alpha-4}{2} + \alpha_-} \text{ or } q = 1 + \frac{\sigma_2}{\frac{m+4}{2} + \beta_-}, \ \ q < 1 + \frac{\sigma_1 + 4 - \alpha}{\frac{n+\alpha-4}{2} + \alpha_-},$$

d) $4 - \alpha + \sigma_1 + \frac{4-\alpha}{4}\sigma_2 > 0$,

$$\max\left\{1, 1 + \frac{\sigma_1}{\frac{n-\alpha+4}{2} + \alpha_-}, \ 1 + \frac{\sigma_2}{\frac{m+4}{2} + \beta_-}\right\} < q \le 1 + \frac{4 - \alpha + \sigma_1 + \frac{4-\alpha}{\alpha}\sigma_2}{\frac{n-\alpha+4}{2} + \alpha_- + \frac{4-\alpha}{4}\left(\frac{m-4}{2} + \beta_-\right)}.$$

Then if $u(x, y)$ *a nontrivial solution of the problem (1.1),(1.2), then* $u \equiv 0$.

Proof. For simplicity of the notation we take $R = 1$. We multiply equation (1.1) by the function $f(x, y) = \xi(x)\eta(y)\varphi(x)\psi(y)$ and integrate with respect to the domain B'. We get the following equality

$$Q \equiv \iint\limits_{B'} |x|^{\sigma_1}|y|^{\sigma_2}|u|^q f dxdy = \iint\limits_{B'} \Delta_x(|x|^\alpha \Delta_x u) f dxdy$$
$$+ \iint\limits_{B'} \Delta_y^2 u f dxdy - \iint\limits_{B'} \frac{C_1}{|x|^{4-\alpha}} u f dxdy - \iint\limits_{B'} \frac{C_2}{|y|^4} u f dxdy.$$

Integrting by parts, we get

$$Q = \int\limits_{S_x}\int\limits_{B'_y} \frac{\partial}{\partial \nu_x} \left(|x|^\alpha \Delta_x u\right) \xi\eta\varphi\psi d\sigma_x dy$$
$$- \iint\limits_{B'} \left(\nabla_x \left(|x|^\alpha \Delta_x u\right), \nabla_x(\xi\varphi)\right)\eta\psi dxdy$$
$$+ \int\limits_{B'_x}\int\limits_{S_y} \frac{\partial}{\partial \nu_y} \left(\Delta_y u\right) \xi\eta\varphi\psi dx d\sigma_y$$
$$- \iint\limits_{B'} (\nabla_y(\Delta_y u), \nabla_y(\eta\psi))\xi\varphi dxdy$$

$$-\iint\limits_{B'} \frac{C_1}{|x|^{4-\alpha}} u\xi\eta\varphi\psi dxdy - \iint\limits_{B'} \frac{C_2}{|y|^4} u\xi\eta\varphi\psi dxdy =$$

$$= \Gamma_x^1 - \Gamma_x^2 + \Gamma_x^3 - \Gamma_x^4 + \Gamma_y^1 - \Gamma_y^2 + \Gamma_y^3 - \Gamma_y^4 +$$
$$+ \iint\limits_{B'} u\Delta_x\left(|x|^\alpha \Delta_x(\xi\varphi)\right)\eta\psi dxdy + \iint\limits_{B'} u\Delta_y^2(\eta\psi)\,\xi\varphi dxdy, \tag{2.3}$$

where $\Gamma_x^i, \Gamma_y^i, i = 1,4$ are apporiate surface integrals on $S_x \times B_y^{'}, B_x^{'} \times S_y$ and σ_x, σ_y are the elements of the surfaces S_x, S_y.

Using the properties of the functions $\xi(x), \eta(y), \varphi(x), \psi(y)$ we show that $(-1)^{i-1}\Gamma_x^i$, $(-1)^{i-1}\Gamma_y^i \le 0$, $i = 1,4$.

Obviously all surface integrals on $S_x(2\rho) \times B_y'$ and $B_x' \times S_y(2\rho^\varepsilon)$ are equal to zero.

$$\Gamma_x^1 = \int\limits_{S_x}\int\limits_{B_y'} \frac{\partial}{\partial\nu_x}\left(|x|^\alpha \Delta_x u\right)\xi\eta\varphi\psi d\sigma_x dy = 0,$$

since $\xi|_{|x|=1} = 0$.

$$\begin{aligned}
\Gamma_x^2 &= \int\limits_{S_x}\int\limits_{B_y'} |x|^\alpha \Delta_x u \frac{\partial(\xi\varphi)}{\partial\nu_x}\eta\psi d\sigma_x dy \\
&= \int\limits_{S_x}\int\limits_{B_y'} |x|^\alpha \Delta_x u \left(\frac{\partial\xi}{\partial\nu_x}\varphi + \xi\frac{\partial\varphi}{\partial\nu_x}\right)\eta\psi d\sigma_x dy \\
&= -\int\limits_{S_x}\int\limits_{B_y'} |x|^\alpha \Delta_x u \frac{\partial\xi}{\partial r}\eta\psi d\sigma_x dy \ge 0,
\end{aligned}$$

by vrite of the condition (1.2) and that on S_x $\xi = 0,\ \frac{\partial\xi}{\partial\nu_x} = -\frac{\partial\xi}{\partial r} \le 0,\ \varphi = 1$.

$$\begin{aligned}
\Gamma_x^3 &= \int\limits_{S_x}\int\limits_{B_y'} \frac{\partial u}{\partial\nu_x}|x|^\alpha \Delta_x(\xi\varphi)\eta\psi d\sigma_x dy \\
&= \int\limits_{S_x}\int\limits_{B_y'} \frac{\partial u}{\partial\nu_x}|x|^\alpha \left(\Delta_x\xi\varphi + 2(\nabla_x\xi, \nabla_x\varphi) + \xi\Delta_x\varphi\right)\eta\psi d\sigma_x dy = 0,
\end{aligned}$$

since on S_x $\xi = 0, \Delta_x\xi = 0, \nabla_x\varphi = 0$.

$$\begin{aligned}
\Gamma_x^4 &= \int\limits_{S_x}\int\limits_{B_y'} u\frac{\partial}{\partial\nu_x}\left(|x|^\alpha \Delta_x(\varphi\xi)\right)\eta\psi d\sigma_x dy \\
&= \int\limits_{S_x}\int\limits_{B_y'} u\left[\alpha|x|^{\alpha-2}(\Delta_x\varphi\xi + 2(\nabla_x\varphi, \nabla_x\xi) + \varphi\Delta_x\xi)(x, \nu_x)\right. \\
&\quad \left. + |x|^\alpha \frac{\partial}{\partial\nu_x}\left(\Delta_x\varphi\xi + 2(\nabla_x\varphi, \nabla_x\xi) + \varphi\Delta_x\xi\right)\right]\eta\psi d\sigma_x dy
\end{aligned}$$

$$= \int\limits_{S_x}\int\limits_{B'_y} u\frac{\partial}{\partial\nu_x}(\Delta_x\xi)\eta\psi d\sigma_x dy \geq 0.$$

Here we use the fact that for $|x| = 1$ $u \geq 0$, the function φ and all its derivatives equal zero, $\Delta_x\xi = 0$ and $\frac{\partial}{\partial\nu_x}\Delta_x\xi \geq 0$.

It the same way , we can show that all $(-1)^{i-1}\Gamma_y^i \leq 0, i = 1, 4$.

As a result , from (2.3) we get the following inequality

$$Q = \iint\limits_{B'} u\Delta_x\left(|x|^\alpha\left(\Delta_x\varphi\xi + 2(\nabla_x\xi, \nabla_x\varphi) + \varphi\Delta_x\xi\right)\right)\eta\psi dxdy$$

$$+ \iint\limits_{B'} u\Delta_y\left(\Delta_y\eta\psi + 2(\nabla_y\eta, \nabla_y\psi) + \eta\Delta_y\psi\right)\xi\varphi dxdy$$

$$- \iint\limits_{B'} \frac{C_1}{|x|^{4-\alpha}}u\xi\eta\varphi\psi dxdy - \iint\limits_{B'} \frac{C_2}{|y|^4}u\xi\eta\varphi\psi dxdy$$

$$= \iint\limits_{B'} u\left(\Delta_x(|x|^\alpha\Delta_x\xi) - \frac{C_1}{|x|^{4-\alpha}}\xi\right)\eta\varphi\psi dxdy + \iint\limits_{B'}\left(\Delta_y^2\eta - \frac{C_2}{|y|^4}\eta\right)\xi\varphi\psi dxdy$$

$$+ \iint\limits_{B'} u[2\left(\nabla_x(|x|^\alpha\Delta_x\xi), \nabla_x\varphi\right) + |x|^\alpha\Delta_x\xi\Delta_x\varphi + 2\Delta_x(|x|^\alpha(\nabla_x\xi, \nabla_x\varphi))$$

$$+ \Delta_x(|x|^\alpha\xi\Delta_x\varphi)]\eta\psi dxdy + \iint\limits_{B'} u[2\left(\nabla_y(\Delta_y\eta), \nabla_y\psi\right)$$

$$+ \Delta_y\eta\Delta_y\psi + 2\Delta_y(\nabla_y\eta, \nabla_y\psi) + \Delta_y(\eta\Delta_y\psi)]\xi\varphi dxdy$$

$$= \iint\limits_{B'} u\eta\psi G_1(\xi, \varphi)dxdy + \iint\limits_{B'} u\xi\varphi G_2(\eta, \psi)dxdy, \tag{2.4}$$

where $G_1(\xi, \varphi), G_2(\eta, \psi)$ are the expressions in the square brackets in the first and second integral. Here we take into acount that $\xi(x)$ is the solution of the equation (2.1), $\eta(x)$ is the solution of equation (2.2). As a result, we have

$$Q \leq \iint\limits_{B'} u\eta\psi G_1(\xi, \varphi)dxdy + \iint\limits_{B'} u\xi\varphi G_2(\eta, \psi)dxdy$$

$$\leq \left(\iint\limits_{B_x(\rho,2\rho)\times B_y(1,2\rho^\varepsilon)} |x|^{\sigma_1}|y|^{\sigma_2}|u|^q\xi\eta\varphi\psi dxdy\right)^{\frac{1}{q}}$$

$$\times \left(\iint\limits_{B_x(\rho,2\rho)\times B_y(1,2\rho^\varepsilon)} \frac{|G_1|^{q'}\eta\psi}{|x|^{\sigma_1(q'-1)}|y|^{\sigma_2(q'-1)}\xi^{q'-1}\varphi^{q'-1}}dxdy\right)^{\frac{1}{q'}}$$

$$+\left(\iint\limits_{B_x(1,2\rho)\times B_y(\rho,2\rho^\varepsilon)} |x|^{\sigma_1}|y|^{\sigma_2}|u|^q\xi\eta\varphi\psi dxdy\right)^{\frac{1}{q}}$$

$$\times\left(\iint\limits_{B_x(1,2\rho)\times B_y(\rho,2\rho^\varepsilon)} \frac{|G_2|^{q'}\xi\varphi}{|x|^{\sigma_1(q'-1)}|y|^{\sigma_2(q'-1)}\eta^{q'-1}\psi^{q'-1}}dxdy\right)^{\frac{1}{q'}}$$

$$\leq\left(\iint\limits_{B'} |x|^{\sigma_1}|y|^{\sigma_2}|u|^q\xi\eta\varphi\psi dxdy\right)^{\frac{1}{q}}\left(J_1^{\frac{1}{q'}}+J_2^{\frac{1}{q'}}\right), \tag{2.5}$$

where $\frac{1}{q}+\frac{1}{q'}=1$ and J_1, J_2 one the second integrals in the first and second summoned.

Hence

$$Q\leq C\left(J_1+J_2\right). \tag{2.6}$$

Estimate the integrals J_1, J_2 separately.

As first consider G_1 and G_2.

$$G_1=2\left(\nabla_x(|x|^\alpha\Delta_x\xi),\nabla_x\varphi\right)+|x|^\alpha\Delta_x\xi\Delta_x\varphi+2\Delta_x\left(|x|^\alpha(\nabla_x\xi,\nabla_x\varphi)\right)$$

$$+\Delta_x\left(|x|^\alpha\xi\Delta_x\varphi\right)=2\frac{d}{dr}\left(r^\alpha\left(\frac{d^2\xi}{dr^2}+\frac{n-1}{r}\frac{d\xi}{dr}\right)\right)\frac{d\varphi}{dr}$$

$$+r^\alpha\left(\frac{d^2\xi}{dr^2}+\frac{n-1}{r}\frac{d\xi}{dr}\right)\left(\frac{d^2\varphi}{dr^2}+\frac{n-1}{r}\frac{d\varphi}{dr}\right)$$

$$+\left(\frac{d^2}{dr^2}+\frac{n-1}{r}\frac{d}{dr}\right)\left(r^\alpha\xi\left(\frac{d^2\varphi}{dr^2}+\frac{n-1}{r}\frac{d\varphi}{dr}\right)\right)$$

$$+2\left(\frac{d^2}{dr^2}+\frac{n-1}{r}\frac{d}{dr}\right)\left(r^\alpha\frac{d\xi}{dr}\frac{d\varphi}{dr}\right)=\left(2\alpha r^{\alpha-1}\left(\frac{d^2\xi}{dr^2}+\frac{n-1}{r}\frac{d\xi}{dr}\right)\right.$$

$$\left.+2r^\alpha\left(\frac{d^3\xi}{dr^3}+\frac{n-1}{r}\frac{d^2\xi}{dr^2}-\frac{n-1}{r^2}\frac{d\xi}{dr}\right)\right)\frac{d\varphi}{dr}$$

$$+r^\alpha\left(\frac{d^2\xi}{dr^2}+\frac{n-1}{r}\frac{d\xi}{dr}\right)\frac{d^2\varphi}{dr^2}+(n-1)r^{\alpha-1}\left(\frac{d^2\xi}{dr^2}+\frac{n-1}{r}\frac{d\xi}{dr}\right)\frac{d\varphi}{dr}$$

$$+\left(\alpha(\alpha-1)r^{\alpha-2}\xi+2\alpha r^{\alpha-1}\frac{d\xi}{dr}+r^\alpha\frac{d^2\xi}{dr^2}\right)\left(\frac{d^2\varphi}{dr^2}+\frac{n-1}{r}\frac{d\varphi}{dr}\right)$$

$$+2\left(\alpha r^{\alpha-1}\xi+r^\alpha\frac{d\xi}{dr}\right)\left(\frac{d^3\varphi}{dr^3}+\frac{n-1}{r}\frac{d^2\varphi}{dr^2}-\frac{n-1}{r^2}\frac{d\varphi}{dr}\right)$$

$$+r^\alpha\xi\left(\frac{d^4\varphi}{dr^4}+\frac{n-1}{r}\frac{d^3\varphi}{dr^3}-2\frac{n-1}{r^2}\frac{d^2\varphi}{dr^2}+2\frac{n-1}{r^3}\frac{d\varphi}{dr}\right)$$

$$+\frac{n-1}{r}\left(\alpha r^{\alpha-1}\xi+r^\alpha\frac{d\xi}{dr}\right)\left(\frac{d^2\varphi}{dr^2}+\frac{n-1}{r}\frac{d\varphi}{dr}\right)$$

$$+(n-1)r^{\alpha-1}\xi\left(\frac{d^3\varphi}{dr^3}+\frac{n-1}{r}\frac{d^2\varphi}{dr^2}-\frac{n-1}{r^2}\frac{d\varphi}{dr}\right)+2\frac{d^2}{dr^2}\left(r^\alpha\frac{d\xi}{dr}\right)\frac{d\varphi}{dr}$$

$$+4\frac{d}{dr}\left(\alpha r^{\alpha-1}\frac{d\xi}{dr}+r^{\alpha}\frac{d^2\xi}{dr^2}\right)\frac{d^2\varphi}{dr^2}+2r^{\alpha}\frac{d\xi}{dr}\frac{d^3\varphi}{dr^3}$$

$$+2\frac{n-1}{r}\left(\alpha r^{\alpha-1}\frac{d\xi}{dr}+r^{\alpha}\frac{d^2\xi}{dr^2}\right)\frac{d\varphi}{dr}+2(n-1)r^{\alpha-1}\frac{d\xi}{dr}\frac{d^2\varphi}{dr^2}$$

$$=\Bigg[2\alpha r^{\alpha-1}\left(\frac{d^2\xi}{dr^2}+\frac{n-1}{r}\frac{d\xi}{dr}\right)+2r^{\alpha}\left(\frac{d^3\xi}{dr^3}+\frac{n-1}{r}\frac{d^2\xi}{dr^2}-\frac{n-1}{r^2}\frac{d\xi}{dr}\right)$$

$$+(n-1)r^{\alpha-1}\left(\frac{d^2\xi}{dr^2}+\frac{n-1}{r}\frac{d\xi}{dr}\right)+\left(\alpha(\alpha-1)r^{\alpha-2}\xi+2\alpha r^{\alpha-1}\frac{d\xi}{dr}+r^{\alpha}\frac{d^2\xi}{dr^2}\right)\frac{n-1}{r}$$

$$-2\frac{n-1}{r^2}\left(\alpha r^{\alpha-1}\xi+r^{\alpha}\frac{d\xi}{dr}\right)+2\,(n-1)\,r^{\alpha-3}\xi+\frac{(n-1)^2}{r^2}\left(\alpha r^{\alpha-1}\xi+r^{\alpha}\frac{d\xi}{dr}\right)$$

$$-\,(n-1)^2\,r^{\alpha-3}\xi+2\left(\alpha(\alpha-1)r^{\alpha-2}\frac{d\xi}{dr}+2\alpha r^{\alpha-1}\frac{d^2\xi}{dr^2}+r^{\alpha}\frac{d^3\xi}{dr^3}\right)$$

$$+\,2\frac{n-1}{r}\left(\alpha r^{\alpha-1}\frac{d\xi}{dr}+r^{\alpha}\frac{d^2\xi}{dr^2}\right)\Bigg]\frac{d\varphi}{dr}+\Bigg[r^{\alpha}\left(\frac{d^2\xi}{dr^2}+\frac{n-1}{r}\frac{d\xi}{dr}\right)$$

$$+\alpha(\alpha-1)r^{\alpha-2}\xi+2\alpha r^{\alpha-1}\frac{d\xi}{dr}+r^{\alpha}\frac{d^2\xi}{dr^2}+\frac{2(n-1)}{r}\left(\alpha r^{\alpha-1}\xi+r^{\alpha}\frac{d\xi}{dr}\right)$$

$$-2(n-1)r^{\alpha-2}\xi+\frac{n-1}{r}\left(\alpha r^{\alpha-1}\xi+r^{\alpha}\frac{d\xi}{dr}\right)+(n-1)^2r^{\alpha-2}\xi$$

$$+4\alpha r^{\alpha-1}\frac{d\xi}{dr}+4r^{\alpha}\frac{d^2\xi}{dr^2}+2(n-1)\,r^{\alpha-1}\frac{d\xi}{dr}\Bigg]\frac{d^2\varphi}{dr^2}$$

$$+\left[2\alpha r^{\alpha-1}\xi+2r^{\alpha}\frac{d\xi}{dr}+2(n-1)r^{\alpha-1}\xi+2r^{\alpha}\frac{d\xi}{dr}\right]\frac{d^3\varphi}{dr^3}+r^{\alpha}\xi\frac{d^4\varphi}{dr^4}. \tag{2.7}$$

Since for $|x|>1$

$$\left|\frac{d^i\xi}{dr^i}\right|\le Cr^{-\frac{n+\alpha-4}{2}+\alpha_{-}-i},\, i=0,1,2,3,4,$$

then from (2.7) we get

$$|G_1|\le Cr^{-\frac{n+\alpha-4}{2}+\alpha_{-}+\alpha-4}\sum_{i=1}^{4}r^i\left|\frac{d^i\varphi}{dr^i}\right|. \tag{2.8}$$

In a similar way, we get

$$|G_2|\le Cr^{-\frac{m-4}{2}+\beta_{-}-4}\sum_{i=1}^{4}r^i\left|\frac{d^i\psi}{dr^i}\right|. \tag{2.9}$$

Using (2.8),(2.9) and making substitution $s=\frac{r}{\rho},\tau=\frac{r}{\rho^{\varepsilon}},\widetilde{\varphi}(s)=\varphi(s\rho),\widetilde{\psi}(\tau)=\psi(\tau\rho^{\varepsilon})$ for J_1,J_2 we get the following estimaties.

$$|J_1|\le\iint\limits_{B_x(\rho,2\rho)\times B_y(1,2\rho^{\varepsilon})}\frac{|G_1|^{q'}\eta\psi}{|x|^{\sigma_1(q'-1)}|y|^{\sigma_2(q'-1)}\xi^{q'-1}\varphi^{q'-1}}dxdy$$

$$\leq \int\limits_{B_x(\rho,2\rho)} \frac{|G_1|^{q'}}{|x|^{\sigma_1(q'-1)}\xi^{q'-1}\varphi^{q'-1}}dx \int\limits_{B_y(1,2\rho^\varepsilon)} \frac{\eta\psi}{|y|^{\sigma_2(q'-1)}}dy$$

$$\leq C\int\limits_{\rho}^{2\rho} \frac{r^{\left(-\frac{n+\alpha-4}{2}+\alpha_-+\alpha-4\right)q'}\left(\sum\limits_{i=1}^{4} r^i\left|\frac{d^i\varphi}{dr^i}\right|\right)^{q'} r^{n-1}}{r^{\sigma_1(q'-1)}r^{\left(-\frac{n+\alpha-4}{2}+\alpha_-\right)(q'-1)}\varphi^{q'-1}}dr$$

$$\times\int\limits_{1}^{2\rho^\varepsilon}\frac{r^{-\frac{m-4}{2}+\beta_-}r^{m-1}}{r^{\sigma_2(q'-1)}}dr \leq c\rho^{(\alpha-4)q'+b}\int\limits_{1}^{2}\frac{s^{\left(-\frac{n+\alpha-4}{2}+\alpha_-+\alpha-4\right)}\left(\sum\limits_{i=1}^{4}s^i\left|\frac{d^i\widetilde{\varphi}}{ds^i}\right|\right)^{q'}}{s^{\sigma_1(q'-1)}s^{\left(-\frac{n+\alpha-4}{2}+\alpha_-\right)(q'-1)-n+1}\widetilde{\varphi}^{q'-1}}ds$$

$$\times\begin{cases} r^a\big|_1^{2\rho^\varepsilon}, & if \quad a\neq 0,\\ \ln r\big|_1^{2\rho^\varepsilon}, & if \quad a=0\end{cases} \leq C\ \widetilde{J}_1\begin{cases}\rho^{(\alpha-4)q'+b+\varepsilon a}, & if\ a>0,\\ \rho^{(\alpha-4)q'+b}\ln 2\rho^\varepsilon, & if\ a=0,\\ \rho^{(\alpha-4)q'+b}, & if\ a<0.\end{cases} \tag{2.10}$$

$$|J_2| \leq \iint\limits_{B_x(1,2\rho)\times B_y(\rho^\varepsilon,2\rho^\varepsilon)} \frac{|G_2|^{q'}\xi\varphi}{|x|^{\sigma_1(q'-1)}|y|^{\sigma_2(q'-1)}\eta^{q'-1}\psi^{q'-1}}dxdy$$

$$\leq \int\limits_{B_x(1,2\rho)}\frac{\xi\varphi}{|x|^{\sigma_1(q'-1)}}dx\int\limits_{B_y(\rho^\varepsilon,2\rho^\varepsilon)}\frac{|G_2|^{q'}}{|y|^{\sigma_2(q'-1)}\eta^{q'-1}\psi^{q'-1}}dy$$

$$\leq C\int\limits_1^{2\rho} r^{-\frac{n+\alpha-4}{2}+\alpha_--\sigma_1(q'-1)+n-1}dr\int\limits_{\rho^\varepsilon}^{2\rho^\varepsilon}\frac{r^{\left(-\frac{m-4}{2}+\beta_--4\right)q'}\left(\sum\limits_{i=1}^{4}r^i\left|\frac{d^i\psi}{dr^i}\right|\right)^{q'}r^{m-1}}{r^{\sigma_2(q'-1)}r^{\left(-\frac{m-4}{2}+\beta_-\right)(q'-1)}\psi^{q'-1}}dr$$

$$\leq C\rho^{\varepsilon(-4q'+a)}\int\limits_1^2\frac{\tau^{\left(-\frac{m-4}{2}+\beta_--4\right)q'}\left(\sum\limits_{i=1}^{4}\tau^i\left|\frac{d^i\widetilde{\psi}}{d\tau^i}\right|\right)^{q'}\tau^{m-1}}{\tau^{\sigma_2(q'-1)}\tau^{\left(-\frac{m-4}{2}+\beta_-\right)(q'-1)}\widetilde{\psi}}d\tau$$

$$\times\begin{cases}\rho^b, & if\ b>0,\\ \ln 2\rho, & if\ b=0,\\ 1, & if\ b<0\end{cases} \leq C\widetilde{J}_2\begin{cases}\rho^{-4\varepsilon q'+\varepsilon a+b}, & if\ b>0,\\ \rho^{-4\varepsilon q'+\varepsilon a}\ln 2\rho, & if\ b=0,\\ \rho^{-4\varepsilon q'+\varepsilon a}, & if\ b<0.\end{cases} \tag{2.11}$$

By $\widetilde{J}_1, \widetilde{J}_2$ we denote the last integral in the inequalities (2.8),(2.9). It is known that under appriate choice of functions φ and ψ the integrals $\widetilde{J}_1, \widetilde{J}_2$ converge (see [15]).

Now we take such ε that

$$-4\varepsilon q' + \varepsilon a + b = (\alpha-4)q' + b + \varepsilon a.$$

Hence

$$-4\varepsilon = (\alpha-4) \Rightarrow \varepsilon = \frac{4-\alpha}{4}.$$

Then, taking into account (2.10),(2.11) in (2.6) we get the followings

$$Q \le \begin{cases} C\rho^{(\alpha-4)q'+b+\frac{4-\alpha}{4}a} & if \quad a>0, \quad b>0, \\ C\rho^{(\alpha-4)q'}\ln\rho & if \quad a=0, \quad b=0, \\ C\rho^{(\alpha-4)q'+\frac{4-\alpha}{4}a}(c_1+c_2\ln\rho) & if \quad a>0, \quad b=0, \\ C\rho^{(\alpha-4)q'+b}\ (c_1+c_2\ln\rho) & if \quad a=0, \quad b>0, \\ C\rho^{(\alpha-4)q'} & if \quad a<0, \quad b<0, \\ C\rho^{(\alpha-4)q'}\ \ln\rho & if \ a=0, b<0 \ or\ a<0, b=0, \\ C\rho^{(\alpha-4)q'+\frac{4-\alpha}{4}a} & if \quad a>0, \quad b<0, \\ C\rho^{(\alpha-4)q'+b} & if \quad a<0, \quad b>0. \end{cases} \tag{2.12}$$

Let us consider separate cases.

1) Let at first $a \le 0, b \le 0$. Since $\alpha - 4 < 0$, then if follows from (2.12) that when tending of $\rho \to +\infty, Q \le 0$.

So,

$$\iint\limits_{B'} |x|^{\sigma_1}|y|^{\sigma_2}|u|^q \xi\eta dxdy \le 0.$$

Hence we get $u \equiv 0$. From the inequality $a \le 0, b \le 0$ it follows

$$\frac{m+4}{2} + \beta_- - \sigma_2(q'-1) \le 0, \quad \frac{n-\alpha+4}{2} + \alpha_- - \sigma_1(q'-1) \le 0.$$

From here

$$q \le 1 + \frac{\sigma_2}{\frac{m+4}{2}+\beta_-}, \quad q \le 1 + \frac{\sigma_1}{\frac{n-\alpha+4}{2}+\alpha_-}.$$

As a result, we get that if $\sigma_1, \sigma_2 > 0$ and $1 < q \le \min\left\{1 + \frac{\sigma_1}{\frac{n-\alpha+4}{2}+\alpha_-}, 1 + \frac{\sigma_2}{\frac{m+4}{2}+\beta_-}\right\}$, then equation (1.1) has no global solutions.

2) Let $a > 0, b \le 0$. This means that

$$q > 1 + \frac{\sigma_2}{\frac{m+4}{2}+\beta_-}, \quad q \le 1 + \frac{\sigma_1}{\frac{n-\alpha+4}{2}+\alpha_-}.$$

From (2.12) we get

$$Q \le C\rho^{(\alpha-4)q'+\frac{4-\alpha}{4}\left(\frac{m+4}{2}+\beta_- - \sigma_2(q'-1)\right)} \text{ for } a>0, b<0, \tag{2.13}$$

$$Q \le C\rho^{(\alpha-4)q'+\frac{4-\alpha}{4}\left(\frac{m+4}{2}+\beta_- - \sigma_2(q'-1)\right)}\ln\rho \text{ for } a>0, b=0. \tag{2.14}$$

If

$$(\alpha-4)q' + \frac{4-\alpha}{4}\left(\frac{m+4}{2} + \beta_- - \sigma_2(q'-1)\right)$$

$$= \left(\alpha - 4 - \sigma_2\frac{4-\alpha}{4}\right)(q'-1) + \frac{4-\alpha}{4}\left(\frac{m+4}{2} + \beta_- - 4\right) < 0,$$

i.e.

$$q < 1 + \frac{4-\alpha+\frac{4-\alpha}{4}\sigma_2}{\frac{4-\alpha}{4}\left(\frac{m+4}{2}+\beta_- - 4\right)} = 1 + \frac{4+\sigma_2}{\frac{m+4}{2}+\beta_- - 4},$$

then when tendig $\rho \to +\infty$ from (2.13), (2.14) we get $Q \leq 0$. Hence, as in the first case we get $u \equiv 0$. What if $q = 1 + \frac{4+\sigma_2}{\frac{m+4}{2}+\beta_- -4}$, then from (2.10),(2.11),(2.12) it follows that $|J_1| \leq C, |J_2| \leq C$ and $Q \leq C$. Tending $\rho \to +\infty$ from (2.13) we have

$$\iint\limits_{B'} |x|^{\sigma_1}|y|^{\sigma_2}|u|^q \xi\eta dxdy \leq C. \tag{2.15}$$

From (2.15) and from the property of the integral if follows, that as $\rho \to +\infty$

$$\iint\limits_{B_x(\rho,2\rho)\times B_y(1,2\rho^\varepsilon)} |x|^{\sigma_1}|y|^{\sigma_2}|u|^q \xi\eta dxdy \to 0, \tag{2.16}$$

$$\iint\limits_{B_x(1,2\rho)\times B_y(\rho^\varepsilon,2\rho^\varepsilon)} |x|^{\sigma_1}|y|^{\sigma_2}|u|^q \xi\eta dxdy \to 0. \tag{2.17}$$

From (2.5) we get that

$$\iint\limits_{B_x(1,\rho)\times B_y(1,2\rho^\varepsilon)} |x|^{\sigma_1}|y|^{\sigma_2}|u|^q \xi\eta dxdy \leq \iint\limits_{B_x(1,2\rho)\times B_y(1,2\rho^\varepsilon)} |x|^{\sigma_1}|y|^{\sigma_2}|u|^q \xi\varphi\eta\psi\, dxdy$$

$$\leq \left(\iint\limits_{B_x(\rho,2\rho)\times B_y(1,2\rho^\varepsilon)} |x|^{\sigma_1}|y|^{\sigma_2}|u|^q \xi\varphi\eta\psi dxdy \right)^{\frac{1}{q}} J_1^{\frac{1}{q'}}$$

$$+ \left(\iint\limits_{B_x(1,2\rho)\times B_y(\rho^\varepsilon,2\rho^\varepsilon)} |x|^{\sigma_1}|y|^{\sigma_2}|u|^q \xi\varphi\eta\psi dxdy \right)^{\frac{1}{q}} J_2^{\frac{1}{q'}}$$

$$\leq C \left(\iint\limits_{B_x(\rho,2\rho)\times B_y(1,2\rho^\varepsilon)} |x|^{\sigma_1}|y|^{\sigma_2}|u|^q \xi\eta dxdy \right)^{\frac{1}{q}}$$

$$+C \left(\iint\limits_{B_x(1,2\rho)\times B_y(\rho^\varepsilon,2\rho^\varepsilon)} |x|^{\sigma_1}|y|^{\sigma_2}|u|^q \xi\eta dxdy \right)^{\frac{1}{q}}.$$

Hence, by virtue of (2.16),(2.17) tending $\rho \to +\infty$ we get

$$\iint\limits_{B'} |x|^{\sigma_1}|y|^{\sigma_2}|u|^q \xi\eta dxdy \leq 0.$$

This means that in this case too $u \equiv 0$. As a result, we got that if $\sigma_1 > 0, \sigma_2 > -4$, $q > \max\left\{1, 1+\frac{\sigma_2}{\frac{m+4}{2}+\beta_-}\right\}$, $q < \left\{1+\frac{\sigma_1}{\frac{n-\alpha+4}{2}+\alpha_-}\right\}$, $q \leq \left\{1+\frac{\sigma_2+4}{\frac{m+4}{2}+\beta_- -4}\right\}$ and $q > \max\left\{1, 1+\frac{\sigma_2}{\frac{m+4}{2}+\beta_-}\right\}$,

$q = \left\{1 + \frac{\sigma_1}{\frac{n-\alpha+4}{2}+\alpha_-}\right\}$, $q < \left\{1 + \frac{\sigma_2+4}{\frac{m+4}{2}+\beta_- -4}\right\}$ the equation (1.1) has no trivial global solutions.

3) Let $a \le 0, b > 0$.This means that

$$q \le 1 + \frac{\sigma_2}{\frac{m+4}{2} + \beta_-}, \;\; q > 1 + \frac{\sigma_1}{\frac{n-\alpha+4}{2} + \alpha_-}.$$

From (2.12) we get

$$Q \le C\rho^{(\alpha-4)q'+b} \text{ for } a < 0, b > 0, \tag{2.18}$$

$$Q \le C\rho^{(\alpha-4)q'+b} \ln \rho \text{ for } a = 0, b > 0. \tag{2.19}$$

If

$$(\alpha-4)q' + b = (\alpha-4)(q'-1) + \alpha - 4 + \\ + \frac{n-\alpha+4}{2} + \alpha_- - \sigma_1(q'-1) < 0,$$

i.e.

$$q < 1 + \frac{\sigma_1 + 4 - \alpha}{\frac{n-\alpha+4}{2} + \alpha_- + \alpha - 4},$$

then tendig $\rho \to +\infty$ from (2.18), (2.19) we get, $Q \le 0$ and respectiely $u \equiv 0$.

Similar in the case 2) we can show that for $a < 0, b > 0$ and in the case $q = 1 + \frac{\sigma_1+4-\alpha}{\frac{n-\alpha+4}{2}+\alpha_-+\alpha-4}$ too $u \equiv 0$.

As a result we get that if $\sigma_2 > 0, \sigma_1 + 4 - \alpha > 0, \;\; q > \max\left\{1, 1 + \frac{\sigma_1}{\frac{n-\alpha+4}{2}+\alpha_-}\right\}$, then at $q < 1+\frac{\sigma_2}{\frac{m+4}{2}+\beta_-}, q \le 1+\frac{\sigma_1+4-\alpha}{\frac{n-\alpha+4}{2}+\alpha_-+\alpha-4}$ or $q = 1+\frac{\sigma_2}{\frac{m+4}{2}+\beta_-}, \;\; q < 1+\frac{\sigma_1+4-\alpha}{\frac{n-\alpha+4}{2}+\alpha_-+\alpha-4}$ equation (1.1) has no non-trivial global solutions.

4) Let now $a > 0, b > 0$.This means that

$$q > 1 + \frac{\sigma_1}{\frac{n-\alpha+4}{2} + \alpha_-}, \;\; q > 1 + \frac{\sigma_2}{\frac{m+4}{2} + \beta_-}.$$

In this case if

$$(\alpha-4)q' + b + \frac{4-\alpha}{4}a = (\alpha-4)(q'-1) + \alpha - 4 \\ + \frac{n-\alpha+4}{2} + \alpha_- - \sigma_1(q'-1) + \frac{4-\alpha}{4}\left(\frac{m+4}{2} + \beta_- - \sigma_2(q'-1)\right) \\ = -\left(4 - \alpha + \sigma_1 + \frac{4-\alpha}{4}\sigma_2\right)(q'-1) + \alpha - 4 \\ + \frac{n-\alpha+4}{2} + \alpha_- + \frac{4-\alpha}{4}\left(\frac{m+4}{2} + \beta_-\right) < 0, \tag{2.20}$$

then tending $\rho \to +\infty$ from (2.12) we get $Q \le 0$. Hence as carlier it follows that $u \equiv 0$. But if $(\alpha-4)q' + b + \frac{4-\alpha}{2}a = 0$, from (2.10),(2.11),(2.12) it follows that $|J_1| \le C, |J_2| \le C, Q \le C$. As in the case 2) from (2.5) we get that in this case also $Q \le 0$ and respectively $u \equiv 0$.

We can write inequality (2.20) in the following form

$$q < 1 + \frac{4 - \alpha + \sigma_1 + \frac{4-\alpha}{4}\sigma_2}{\alpha - 4 + \frac{n-\alpha+4}{2} + \alpha_- + \frac{4-\alpha}{4}\left(\frac{m+4}{2} + \beta_-\right)}.$$

As a result, we get that if $4 - \alpha + \sigma_1 + \frac{4-\alpha}{4}\sigma_2 > 0$,

$$\max\left\{1 + \frac{\sigma_1}{\frac{n-\alpha+4}{2} + \alpha_-}, 1 + \frac{\sigma_2}{\frac{m+4}{2} + \beta_-}\right\}$$

$$< q \leq 1 + \frac{4 - \alpha + \sigma_1 + \frac{4-\alpha}{4}\sigma_2}{\alpha - 4 + \frac{n-\alpha+4}{2} + \alpha_- + \frac{4-\alpha}{4}\left(\frac{m+4}{2} + \beta_-\right)},$$

then equation (1.1) has no non-trivial global solutions.

This completly prines the theorem.

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