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MECHANICS

The Verifying Authenticity And Condition Of Applicability Of The Equation Of The State Of Bubble Liquid

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Abstract. *In this article the calculations were carried out to verify authenticity and condition of applicability of the equation of the state of gas-liquid surround. The dependence of the dimensionless radius of the air bubble, of the pressure in the bubble, of gas temperature on the dimensionless time and analogical dependence for the case when the pressure went down abruptly at room temperature in the water depicted. The results showed that the derived equations of state agree well with R.I. Nigmatulin's formula.*

Keywords. pressure · gas bubble · equation of state · gas-liquid mixture · temperature · volume concentration

In the paper [1], R.I. Nigmatulin has derived the equation of state of a gas-liquid mixture.

Dynamics of single soluble gas bubbles in liquid were considered in detail in the papers [2-5].

Based on the system of equations [2] describing the dynamics of single gas bubbles (we will ignore the gas solubility) we analyse the authenticity and condition of applicability of R.I. Nigmatulin's equation of state of gas-liquid medium. Write in the dimensionless form the system of equations within a two-temperature model with two pressure models describing the dynamics of insoluble gas bubble in liquid [2-4]:

$$\frac{d\theta}{d\tau} = \frac{3\theta}{Y_1 P_2}\left[\gamma\left(1+S\right)G - \left(\gamma-1\right)P_2\dot{Y}_1\right], \quad \frac{dP_2}{d\tau} = \frac{3\gamma}{Y_1}\left[\left(1+S\right)G - P_2\dot{Y}_1\right],$$

$$\frac{dY_1}{d\tau} = Y_2, \quad \frac{dY_2}{d\tau} = -\frac{3}{2}\cdot\frac{Y_2^2}{Y_1} + \frac{P_2 - P_c - S/Y_1}{Y_1}\cdot Pe_2^2 - L\cdot\frac{Y_2}{Y_1^2} \tag{1}$$

$$G = sign\left(1-\theta\right)\cdot\sqrt{\frac{3\left(\gamma-1\right)\theta}{Y_1}\left|\dot{Y}_1\left(1-\theta\right)\right|}, \quad Y_1 = R/R_0, \quad P_2 = p_2/p_0,$$

$$P_c = p_c/p_0, \theta = T_2/T_0, \; Y_2 = \dot{R}t_0/R_0, \; \tau = t/t_0, t_0 = R_0^2/a_2,$$

$$Pe_2 = \frac{R_0}{a_2}\sqrt{\frac{p_0}{\rho_1^0}}, S = 2\sigma/R_0 p_0, \quad L = 4\nu_1/a_2.$$

For $\tau = 0 : \theta = 1, P_2 = 1 + S, \; Y_1 = 1, \; Y_2 = 0.$

Here $p_2\left(t\right)$ is the gas pressure in the bubble of radius $R\left(t\right)$, p_c is the liquid pressure far from the bubble, T_2 is the gas temperature, a_2 is gas heatconductivity, σ is surface tension, ν_1 is kinetic viscosity of liquid, ρ_1^0 is the density of liquid, γ is adiabatic exponent.

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Follow the dependence of mean pressure of gas-fluid mixture $p/p_0 \approx (1-\alpha_2)(p_2/p_0 - 2\sigma/p_0 R)$ on density $p/p_0 \approx (1-\alpha_2)/(1-\alpha_{20})$. In the dimensionless form

$$P \approx (1-\alpha_2)(P_2 - S/Y_1) \quad \Re = \rho/\rho_0 \approx (1-\alpha_2)/(1-\alpha_{20}). \tag{2}$$

For giving volumetric gas content α_2 in equation (2), we use the cellular model (fig. 1)

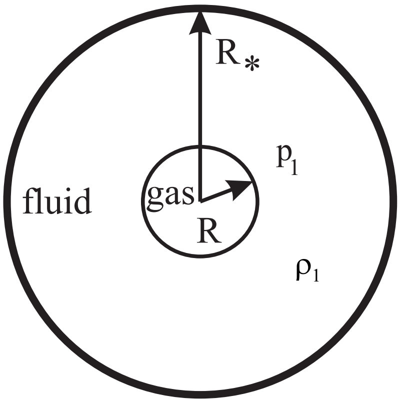

Fig.1.

The volumetric gas content α_2 equals the ratis of total gas volume to the volume of the whole liquid, therefore we get:

$$\alpha_2 = \frac{V_{gas}}{V_{mixture}} = \left(\frac{R}{R_*}\right)^3 = \alpha_{20}\left(\frac{R}{R_0}\right)^3 = \alpha_{20} Y_1^3, \alpha_{20} = \left(\frac{R_0}{R_*}\right)^3,$$

where R_* denotes the radius equivalent to the cell around the bubble [1].

The expression for the mean value of liquid pressure by the volume of the cell was obtained in [1]

$$p_c = p_2 - \frac{2\sigma}{R} - 4\mu_1\frac{\dot{R}}{R} - \rho_1^0\left[(1-\varphi_1) R\ddot{R} + \frac{3}{2}(1-\varphi_2)\dot{R}^2\right],$$

$$\varphi_1 = \frac{3}{2}\frac{\alpha_2^{1/3} - \alpha_2}{1-\alpha_2}, \varphi_2 = \frac{\alpha_2^{1/3}(\alpha_2+2) - 3\alpha_2}{1-\alpha_2}. \tag{3}$$

In the system of equations (1), the Nusselt parameter was given in the form obtained in the paper [1]:

$$Nu_2 = \sqrt{\frac{12(\gamma-1)TR}{a_2}\left|\frac{\dot{R}}{T_0 - T_2}\right|}.$$

The system of equations (1) is a closed system of equations describing dynamics and heat exchange of insoluble gas bubble with liquid. We write the initial conditions in the form: for $\tau = 0 : \theta = 1$, $P_2 = 1 + S$, $Y_1 = 1$, $Y_2 = 0$. We get the Cauchy problem for the system of differential equations (1).

We write the system of equations (1) in finite differences having denoted by h the time integration step, and by P_{2i}, θ_i, Y_{1i} and Y_{2i} the appropriate values of variables at partition points of the time segment. The we get:

$$\theta_0 = 1,\ P_{20} = 1 + S,\ \ Y_{10} = 1,\ \ Y_{20} = 0. \tag{4}$$

$$\theta_i = \theta_{i-1} + \frac{3\theta_{i-1}h}{Y_{1i-1}P_{2i-1}}\left[\gamma(1+S)G_{i-1} - (\gamma-1)P_{2i-1}Y_{2i-1}\right],\ i = 1, 2, ... \tag{5}$$

$$P_{2i} = P_{2i-1} + \frac{3\gamma h}{Y_{1i-1}}\left[(1+S)G_{i-1} - P_{2i-1}Y_{2i-1}\right],\ i = 1, 2, ... \tag{6}$$

$$G_{i-1} = sign\left(1-\theta_{i-1}\right)\cdot\sqrt{\frac{3\left(\gamma-1\right)\theta_{i-1}}{Y_{1i-1}}\left|Y_{2i-1}\left(1-\theta_{i-1}\right)\right|}$$

$$Y_{1i} = Y_{i1-1} + hY_{i2-1} \tag{7}$$

$$Y_{2i} = Y_{2i-1} - \frac{3h}{2}\cdot\frac{Y_{2i-1}^2}{Y_{1i-1}} + \frac{P_{2i-1}-P_c-S/Y_{1i-1}}{Y_{1i-1}}\cdot Pe_2^2 h - L\cdot\frac{hY_{2i-1}}{Y_{1i-1}^2}. \tag{8}$$

Let us consider a problem of radial motion of a bubble arising at instant change of pressure for $\tau = 0$, in liquid far from the bubble with p_0 to p_c $\quad \varDelta p = p_c - p_0$.

We take the necessary thermophysical characteristics of air and water at atmospheric pressure $p_0 = 10^5 n/m^2$ and room temperature $T_0 = 293^0 K$, equal to [7]:

$$\rho_1^0 = 963\frac{kg}{m^3}, \sigma = 0,06\frac{n}{m}, \mu_1 = 2,7\cdot 10^{-4}\frac{n\ \text{sec}.}{m}, \nu_1 = \mu_1/\rho_1^0 = 3\cdot 10^{-7}\frac{m^2}{\text{sec}.}$$

$$c_{p2} = 1000\frac{J}{kg\cdot degree}, c_{V2} = 714,3\frac{J}{kg\cdot degree}, \lambda_2 = 0,0262\frac{W}{m\cdot degree},$$

$$\rho_{20} = 1,16\frac{kg}{m^3},\ \ \gamma = 1,34,\ \ a_2 = 2,26\cdot 10^{-5}\frac{m^2}{\text{sec}},\ \ L = 4\nu_1/a_2 = 5,3\cdot 10^{-2},$$

$$Pe_2 = \frac{R_0}{a_2}\sqrt{\frac{p_0}{\rho_1^0}} = 4,6\cdot 10^5 R_0.$$

Control of account was the degree of conservation of gas mass in the bubble m_2:

$$\frac{d}{dt}\left(\frac{4}{3}\pi R^3\rho_2^0\right) = 0 \quad \text{or} \quad \frac{Y_1^3 P_2}{\theta} = 1 + S. \tag{9}$$

Choice of the step of finite-difference grid h was selected from the condition of accuracy of fulfilment of condition (9) and was accepted to be equal to $h = 0,001$. In all calculations this condition was fulfilled within to 1%.

The dependences of dimensionless radius of air bubble (curve 1) and air pressure in the bubble (curve 3) and gas temperature (curve 2) on dimensionless time when the pressure in water abruptly increased at room temperature $T_0 = 293^0 K$ from $p_0 = 1at$ to $p_c = 1,5at$ were depicted in fig. 2 . The initial radius of the bubble in rest was accepted to be equal to 10 mkm, and the initial radial velocity of pulsations was accepted to be equal to zero.

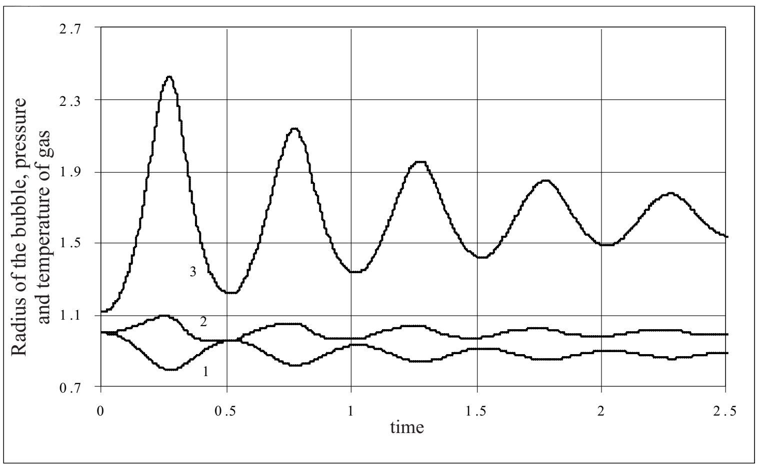

Fig.2.

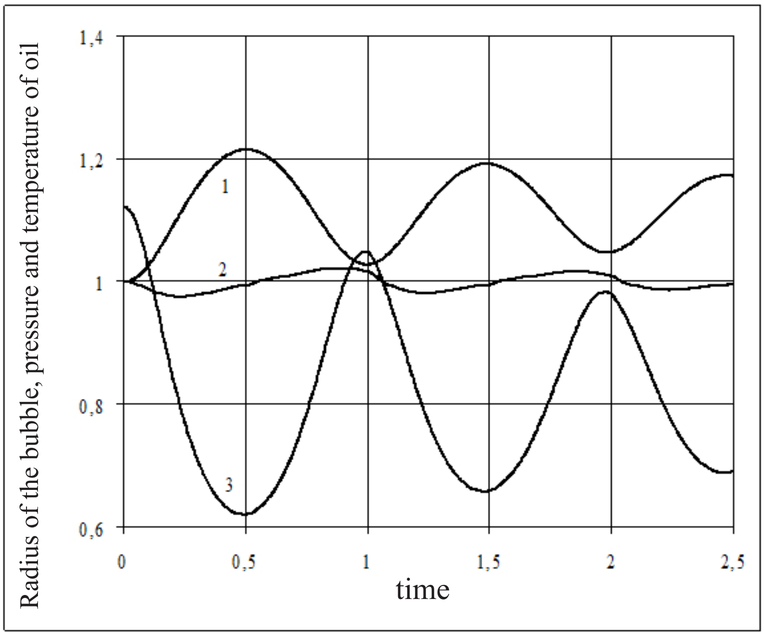

Fig.3.

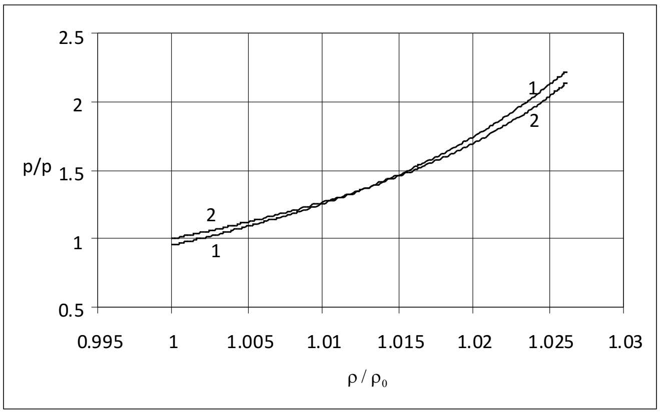

Fig.4.

Similar dependences are given in fig. 3 for the case when the pressure in water abruptly decreased at room temperature $T_0 = 293^0K$ from $p_0 = 1at$ to $p_c = 0,7at$.

We compare the results with the equation of state of gas-liquid mixture [1] with incompressible carrier phase with regard to surface tension:

$$\frac{p}{p_0} = \frac{\alpha_{20}\,(1+S)\,\rho/\rho_0}{1-\alpha_{10}\rho/\rho_0} - S \cdot \sqrt[3]{\frac{\alpha_{20}\rho/\rho_0}{1-\alpha_{10}\rho/\rho_0}}. \tag{10}$$

The results of calculations by the results of numerical solution of the system of equations (5)-(8) and formula (2) (curve 2) and by formula (10) for variants of calculations of fig. 2 and 3 are depicted in figures 4 and 5. Behavior of the curves in fig. 4 and 5 shows that the calculations of the present paper agree well with R.I. Nigmatulin's formula (curves 1).

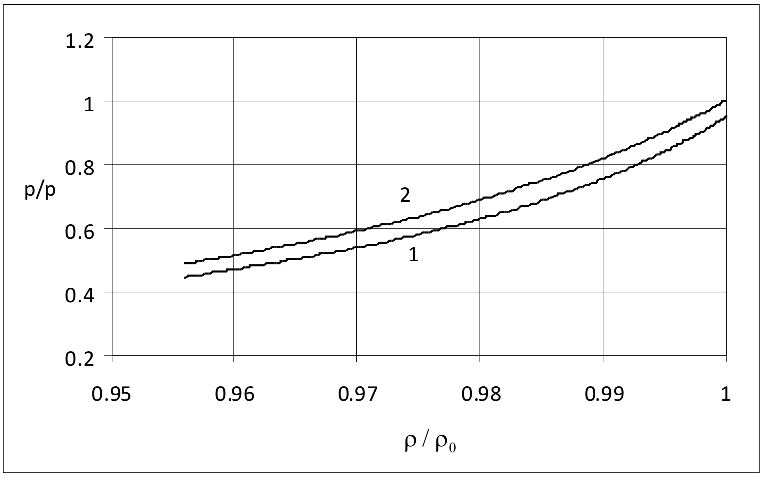

Fig.5.

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Legend of figures

Fig. 1. Cell of a liquid with bubble

Fig. 2. Dependences of the radius of air bubble, pressure and air temperature in the bubble on time at pressure increase in liquid.

Fig. 3. Dependences of radius of air bubble, pressure and temperature in the bubble on time at pressure decrease in liquid.

Fig. 4. Comparison of the calculation results with the equation of state of R.I. Nigmatulin.

Fig. 5. Comparison of the calculation results with the equation of state of R.I. Nigmatulin.