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ON AN INVERSE BOUNDARY VALUE PROBLEM WITH TIME NONLOCAL CONDITIONS FOR ONE-DIMENSIONAL HYPERBOLIC EQUATION

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Abstract. In the paper we consider an one -dimensional hyperbolic equation with nonlocal initial data of integral form is considered, and the existence of a unique classical solution is proved.

Keywords. inverse boundary value problem, hyperbolic equation, nonlocal conditions, classical solution.

Mathematics Subject Classification (2010): 35J25; 35R30

1 Introduction

There a lot of cases when the needs of practice leads to problems of definition of the coefficients or the right hand side of a differential equation according to some data from its solution. Such problems received the name of inverse problems of mathematical physics. The inverse problems is an actively developing second of contemporary mathematics. At present, theory of nonlocal problems is intensively developing and is an important section of partial differential equations. In this field, the problem with nonlocal integral conditions are of great interest. Note that in a great majority of publications devoted to problems with nonlocal integral conditions for hyperbolic equations, spatial nonlocal are considered [1, 4, 5]. In the paper [2], a problem with time nonlocal integral conditions considered for a hyperbolic equation. In the suggested paper, an inverse boundary value problem with time nonlocal integral conditions is considered for a hyperbolic equation.

2 Problem statement

Consider for the equation

$$u_{tt}(x,t) - u_{xx}(x,t) = a(t)u(x.t) + f(x,t) \tag{2.1}$$

in domain $D_T = \{(x,t) : 0 \le x \le 1,\ 0 \le t \le T\}$ an inverse problem with the conditions

$$u(x,0) = \varphi(x),$$

$$u_t(x,0) = \psi(x) + \int_0^T M(t)u(x,t)dt \quad (0 \le x \le 1), \tag{2.2}$$

$$u(0,t) = u_x(1,t) = 0 \quad (0 \le t \le T), \tag{2.3}$$

and with an additional condition

$$u(1,t) = h(t) \quad (0 \le t \le T), \tag{2.4}$$

where $f(x,t),\ \varphi(x),\ \psi(x), M(t),\ h(t)$ are the given functions, $u(x,t)$ and $a(t)$ are the sought functions.

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Definition 2.1 *Under the classical solution of the inverse boundary value problem* (2.1)-(2.4) *we will understand the pair* $\{u(x,t), a(t)\}$ *of functions* $u(x,t)$ *and* $a(t)$, *if* $u(x,t) \in C^2(D_T)$, $a(t) \in C[0,T]$ *and relations* (2.1)-(2.4) *are fulfilled in the ordinary sense.*

Consider the problem

$$y''(t) = a(t)y(t) \quad (0 \le t \le T), \tag{2.5}$$

$$y(0) = 0, \;\; y'(0) = \int_0^T M(t)y(t)dt, \tag{2.6}$$

where $a(t), M(t) \in C[0,T]$ are the given functions, $y = y(t)$ is the sought function.

We prove the following

Lemma 2.1 *Let* $M(t) \in C[0,T]$, $a(t) \in C[0,T]$ *such that*

$$\|a(t)\|_{C[0,T]} \le R \equiv const. \tag{2.7}$$

Furthermore,

$$\left(\|M(t)\|_{C[0,T]} + \frac{1}{2}R\right)T^2 < 1. \tag{2.8}$$

Then problem (2.5),(2.6) *has only a trivial solution.*

Proof. It is easy to see that problem (2.5), (2.6) is equivalent to the integral equation

$$y(t) = t\int_0^T M(t)y(t)dt + \int_0^t (t-\tau)a(\tau)y(\tau)d\tau. \tag{2.9}$$

Having denoted

$$Ay(t) = t\int_0^T M(t)y(t)dt + \int_0^t (t-r)a(\tau)y(\tau)d\tau, \tag{2.10}$$

we write (2.9) in the from

$$y(t) = Ay(t). \tag{2.11}$$

We will investigate equation (2.11) in the space $C[0,T]$

It is easy to see that the operator A is continuous in the space $C[0,T]$

Show that the operator A is contractive in the space $C[0,T]$. Indeed, for any $C[0,T]$ from the space we have $y(t), \bar{y}(t)$:

$$\|Ay(t) - A\bar{y}(t)\|_{C[0,T]} \le \left(\|M(t)\|_{C[0,T]} + \frac{1}{2}R^2\right)T^2 \|y(t) - \bar{y}(t)\|_{C[0,T]}\,. \tag{2.12}$$

From (2.12), allowing for (2.8) it follows that the operator A is contractive in the space $C[0,T]$. Therefore, in the space $C[0,T]$ the operator A has a unique fixed point $y(t)$ that is a unique solution of equation (2.11). Thus, in $C[0,T]$ integral equation (2.9) has a unique solution, consequently problem (2.5), (2.6) has also in $C[0,T]$ a unique solution. As $y(t) = 0$ is the solution of problem (2.5), (2.6) then it has only a trivial solution.

The lemma is proved.

Now let's consider the following auxiliary inverse boundary value problem. It is required to determine the pair $\{u(x,t), a(t)\}$ of functions $\bar{u}(x,t) \in C^2(D_1)$, $a(t) \in C[0,T]$ from (2.1)-(2.3) and

$$h''(t) - u_{xx}(1,t) = a(t)h(t) + f(1,t) \quad (0 \le t \le T). \tag{2.13}$$

The following theorem is valid.

Theorem 2.1 *Let* $\varphi(x), \psi(x) \in C[0,T]$, $M(t) \in C[0,T]$, $h(t) \in C^2[0,T]$ $(0 \le t \le T)$, $f(x,t) \in C(D_T)$, *and*

$$\varphi(1) = h(0), \quad \psi(1) + \int_0^T M(t)h(t)dt = h'(0)$$

the argument condition be fulfilled.

Then the following statements are valid

1).Each classical solution $\{u(x,t), a(t)\}$ *of problem* (2.1)-(2.4) *is the solution of problem* (2.1)-(2.3), (2.13) *as well;*

2).Each solution $\{u(x,t), a(t)\}$ *of problem* (2.1)-(2.3), (2.13) *is such that is the classical solution of problem* (2.1)-(2.4)

$$\left(\|M(t)\|_{C[0,T]} + \frac{1}{2}\|a(y)\|_{C[0,T]}\right)T^2 < 1. \tag{2.14}$$

Proof. Let $\{u(x,t), a(t)\}$ be the solution of problem (2.1)-(2.4). Assuming $h(t) \in C^2[0,T]$ and twice differentiating (2.4), we get:

$$u_t(1,t) = h'(t), \;\; u_{tt}(t) = h''(t) \quad (0 \le t \le T). \tag{2.15}$$

Plugging $x = 1$ into equation (2.1), we have

$$u_{tt}(1,t) = u_{xx}(1,t) = a(t) \;\; u(1,t) + f(1,t) \quad (0 \le t \le T). \tag{2.16}$$

Hence, allowing for (2.4) and (2.15) we arrive to fulfillment of (2.13).

Now, assuming that $\{u(x,t), a(t)\}$ is the solution of problem (2.1)-(2.3), (2.13). Then from (2.13) and (2.16), we find

$$\frac{d^2}{dt^2}\left(u(1,t) - h(t)\right) = a(t)\left(u(1,t) - h(t)\right) \quad (0 \le t \le T) \tag{2.17}$$

by virtue of (2.2) and $\varphi(1) = h(0)$, $\psi(1) + \int_0^T M(t)h(t)dt = h'(0)$ it is obvious that,

$$u(1,0) - h(0) = \varphi(1) - h(0) = 0,$$

$$u_t(1,0) - h'(0) - \int_0^T M(t)(u(1,t) - h(t))dt = \psi(1) + \int_0^T M(t)h(t) - h'(0) = 0. \tag{2.18}$$

As, by virtue of lemma 1, problem (2.17), (2.18) has only a trivial solution, then $u(1,t) - h(t) = 0$, i.e. condition (2.4) is fulfilled.

The theorem is proved.

3 Investigate existence and uniqueness of the classical solution of the inverse boundary value problem.

We will seek for the solution $u(x,t)$ $\{u(x,t), a(t)\}$ of problem (2.1)-(2.3), (2.13) in the form

$$u(x,t) = \sum_{k=1}^{\infty} u_k(t)\sin\lambda_k x \quad \left(\lambda_k = \frac{\pi}{2}(2k-1)\right), \tag{3.1}$$

where

$$u_k(t) = 2\int_0^1 u(x,t)\sin\lambda_k x dx \quad (k = 1, 2)$$

is twice continuously differentiable functions on the segment $[0,T]$. Then, using formal scheme of Fourier method from (2.1) and (2.2) have

$$u_k''(t) + \lambda_k^2 u_k(t) = F_k(t;u,a) \; (k = 1, 2...; 0 \le t \le T), \tag{3.2}$$

$$u_k(0) = \varphi_k,$$

$$u_k^{'}(0) = \psi_k + \int_0^T M(t)u_k(t)dt \quad (k = 1, 2...0 \le t \le T), \tag{3.3}$$

where

$$F_k(t; u, a) = f_k(t) + a(t)u_k(t), \quad f_k(t) = 2\int_0^1 f(x,t)\sin\lambda_k x dx,$$

$$\varphi_k = 2\int_0^1 \varphi(x)\sin\lambda_k t + \quad \psi_k = 2\int_0^1 \psi(x)\sin\lambda_k x dx \quad (k = 1, 2...).$$

By solving problem (3.2), (3.3) we find

$$u_k(t) = \varphi_k \cos\lambda_k t + \frac{1}{\lambda_k}\left(\psi_k + \int_0^T M(t)u_k(t)dt\right)\sin\lambda_k t$$

$$+\frac{1}{\lambda_k}\int_0^t F_k(\tau; u, a)\sin\lambda_k(t-\tau)d\tau \quad (k = 1, 2, ...). \tag{3.4}$$

For determining the first component $u(x,t)$ of the solution of problem (2.1)-(2.3), (2.13), taking into account relation (3.4), from (3.1) we get

$$u(x,t) = \sum_{k=1}^{\infty}\left\{\varphi_k \cos\lambda_k t + \frac{1}{\lambda_k}\left(\psi_k + \int_0^T M(t)u_k(t)dt\right)\sin\lambda_k t\right.$$

$$\left.+\frac{1}{\lambda_k}\int_0^T F(\tau; u, a)\sin\lambda_k(t-\tau)d\tau\right\}\sin\lambda_k x. \tag{3.5}$$

Now, from (2.13), allowing for (3.1), we get

$$a(t) = h^{-1}(l)\left\{h''(t) - f(1,t) + \sum_{k=1}^{\infty}(-1)^{k+1}\lambda_k^2 u_k(t)\right\}. \tag{3.6}$$

In order to obtain an equation for the second component $a(t)$ of the solution $\{u(x,t), a(t)\}$ of problem (2.1)-(2.3), (2.13) we substitute the expression (3.4) into (3.6):

$$a(t) = h^{-1}(t)\left\{h^n(t) - f(1,t) + \sum_{k=1}^{\infty}\lambda_k^2(-1)^{k+1}\left[\varphi_k\cos\lambda_k t\right.\right.$$

$$\left.\left.+\frac{1}{\lambda_k}\left(\psi_k + \int_0^T M(t)u_k(t)dt\right)\sin\lambda_k x + \frac{1}{\lambda_k}\int_0^t F(\tau; u, a)\sin\lambda_k(t-\tau)dt\right]\right\}. \tag{3.7}$$

Thus, the solution of problem (2.1)-(2.3), (2.13) was reduced to the solution of system (3.5), (3.7) with respect to unknown function $u(x,t)$ and $a(t)$. For studying the uniqueness of the solution of problem (2.1)-(2.3), (2.13) the following lemma is of great importance.

Lemma 3.1 *If* $\{u(x,t), a(t)\}$ *is any classic solution of problem* (2.1)-(2.3), (2.13)*, then the functions* $u_k(t) = 2\int_0^1 u(x,t)\sin\lambda_k x dx$ $(k = 1, 2, ...)$ *satisfy system* (3.4).

Proof. Let $\{u(x,t), a(t)\}$ side be any solution of problem (2.1)-(2.3), (2.13). Then having multiplied the both parts of the equation by the function $2\sin\lambda_k x (k = 1, 2, ...)$, integrating the obtained inequality with respect to x from zero to unit, and using the relation

$$2\int_0^1 u_{tt}(x,t)\sin\lambda_k x dx = \frac{d^2}{dx^2}\left(2\int_0^1 u(x,t)\sin\lambda_k x dx\right) = u_k''(t),$$

$$2\int_0^1 u_{xx}(x,t)\sin\lambda_k x dx = -\lambda_k^2\left(2\int_0^1 u(x,t)\sin\lambda_k x dx\right) = -\lambda_k^2 u_k(t),$$

$k = 1, 2, ...$, we get that equation (3.2) is satisfied.

In the same way from (2.2) we get that condition (3.3) is fulfilled.

Thus, $u_k(t) (k = 1, 2, ...)$, is the solution of problem. Hence it directly follows that the functions (3.2), (3.3) satisfy system $u_k(t)$ $(k = 1, 2, ...)$.

The lemma is proved.

Remark 3.1 From lemma 2 it follows that for proving the uniqueness of the solution of problem (2.1)-(2.3), (2.13) it suffices to prove uniqueness of the solution of problem (3.5), (3.7).

For studying problem (3.5),(3,7) we consider the following spaces.
Denote by $B_{2,T}^3$ [3] the set of all functions $u(x,t)$ of the form

$$u(x,t)=\sum_{k=1}^{\infty}u_k(t)\sin\lambda_k x,\ \left(\lambda_k=\frac{\pi}{2}(2k-1)\right).$$

Considered in D_T, where each of the functions $u_k(t)$ $(k=1,2,...)$, is continuous on $[0,T]$,

$$I_T(u)=\left(\sum_{k=1}^{\infty}\left(\lambda_k^3\left\|u_k(t)\right\|_{C[0,T]}\right)^2\right)^{\frac{1}{2}}<+\infty.$$

We define the norm in this set in the form $\|u(x,t)\|_{B_{2,T}^3}=I_T(u)$.
Denote by E_T^3 a space consisting of topological product $B_{2,T}^3\times C[0,T]$.
The norm of the element $z=\{u,a\}$ is determined by the formula

$$\|z\|_{E_T^3}=\|u(x,t)\|_{B_{2,T}^3}+\|a(t)\|_{C[0,T]}.$$

It is known that $B_{2,T}^3$ and E_T^3 are Banach spaces.
Now consider in the space E_T^3 operator

$$\Phi(u,a)=\{\Phi_1(u,a),\Phi_2(u,a)\},$$

where

$$\Phi_1(u,a)=\tilde u(x,t)=\sum_{k=1}^{\infty}\tilde u(t)\sin\lambda_k x,\ \Phi_2(u,a)=\tilde a(t),$$

$\tilde u_k(t)(k=1,2,...)$ and $\tilde a(t)$ are equal to right sides of (3.4) and (3.7) respectively.
It is easy to see that

$$\left\{\sum_{k=1}^{\infty}\left(\lambda_k^3\left\|\tilde u_k(t)\right\|_{C[0,T]}\right)^2\right\}^{\frac{1}{2}}\le\sqrt{5}\left(\sum_{k=1}^{\infty}\left(\lambda_k^3\left|\varphi_k\right|\right)^2\right)^{\frac{1}{2}}$$

$$+\sqrt{5}\left(\sum_{k=1}^{\infty}\left(\lambda_k^2\left|\psi_k\right|\right)^2\right)^{\frac{1}{2}}+\sqrt{5}\left\|M(t)\right\|_{C[0,T]^T}\left(\sum_{k=1}^{\infty}\left(\lambda_k^3\left\|u_k(t)\right\|_{C[0,T]}\right)^2\right)^{\frac{1}{2}}$$

$$+\sqrt{5T}\left(\int_0^T\sum_{k=1}^{\infty}\left(\lambda_k^2\left|f_k(\tau)\right|\right)^2d\tau\right)^{\frac{1}{2}}+\sqrt{5}T\left\|a(t)\right\|_{C[0,T]^T}\left(\sum_{k=1}^{\infty}\left(\lambda_k^3\left\|u_k(t)\right\|_{C[0,T]}\right)^2\right)^{\frac{1}{2}},\quad(3.8)$$

$$\left\|\tilde a(t)\right\|_{C[0,T]}\le\left\|h^{-1}(t)\right\|_{C[0,T]}\left\{\left\|h''(t)-f(1,t)\right\|_{C[0,T]}+\left(\sum_{k=1}^{\infty}\lambda_k^{-2}\right)^{\frac{1}{2}}\left[\left(\sum_{k=1}^{\infty}\left(\lambda_k^3\left|\varphi_k\right|\right)^2\right)^{\frac{1}{2}}\right.\right.$$

$$+\left(\sum_{k=1}^{\infty}\left(\lambda_k^2\left|\psi_k\right|\right)^2\right)^{\frac{1}{2}}+T\left\|M(t)\right\|_{C[0,T]}\left(\sum_{k=1}^{\infty}\left(\lambda_k^3\left\|u(t)\right\|_{C[0,T]}\right)^2\right)^{\frac{1}{2}}$$

$$\left.\left.+\sqrt{T}\left(\int_0^T\sum_{k=1}^{\infty}\left(\lambda_k^2\left|f_k(\tau)\right|\right)^2d\tau\right)^{\frac{1}{2}}+T\left\|a(t)\right\|_{C[0,T]}\left(\sum_{k=1}^{\infty}\left(\lambda_k^3\left\|u_k(t)\right\|_{C[0,T]}\right)^2\right)^{\frac{1}{2}}\right]\right\}.\quad(3.9)$$

Suppose that the data of problem (2.1)-(2.3), (2.13) satisfy the following conditions

1. $\varphi(x)\in C^2[0,1],\ \varphi'''(x)\in L_2(0,1),\ \varphi(0)=\varphi'(1)=\varphi''(0)=0.$

2. $\psi(x) \in C^1[0,1],\ \psi'''(x) \in L_2(0,1),\ \psi(0) = \psi'(1) = 0.$
3. $f(x,t), f_x(x,t) \in C(D_T),\ f_{xx}(x,t) \in L_2(D_T),$

$$f(0,t) = f_x(1,t) = 0 \quad (0 \le t \le T)$$

4. $M(t) \in C[0,T],\ h(t) \in C^2[0,T],\ h(t) \neq 0\,(0 \le t \le T).$

Then from (3.8) and (3.9) we have:

$$\|\tilde{u}(x,t)\|_{B^3_{2,T}} \le A_1(T) + B_1(T)\|u(x,t)\|_{B^3_{2,T}}\left(\|a(t)\|_{C[0,T]} + 1\right), \tag{3.10}$$

$$\|\tilde{a}(t)\|_{C[0,T]} \le A_2(T) + B_2(T)\|u(x,t)\|_{B^3_{2,T}}\left(\|a(t)\|_{C[0,T]} + 1\right), \tag{3.11}$$

where

$$A_1(T) = \sqrt{5}\left\|\varphi'''(x)\right\|_{L_2(0,1)} + \sqrt{5}\left\|\psi''(x)\right\|_{L_2(0,1)} + \sqrt{5}\left\|f_{xx}(x)\right\|_{L_2(0,T)},$$

$$B_1(T) = \sqrt{5}\left(\|M(t)\|_{C[0,T]} + 1\right)T,$$

$$A_2(T) = \left\|h^{-1}(t)\right\|_{C[0,T]} + \left\{\left\|h''(t) - f(l,t)\right\|_{C[0,T]}\left(\sum_{k=1}^{\infty}\lambda_k^{-2}\right)^{1/2}\left[\left\|\varphi'''(x)\right\|_{L_2(0,1)}\right.\right.$$
$$\left.\left. + \left\|\varphi''(x)\right\|_{L_2(0,1)}\right] + \|f_{xx}(x)\|_{L_2(D_T)}\right]\Big\}$$

$$B_2(T) = \left\|h^{-1}(t)\right\|_{C[0,T]}\left(\|M(t)\|_{C[0,T]} + 1\right)T.$$

From inequalities (3.10), (3.11) we conclude that

$$\|\tilde{u}(x,l)\|_{B^3_{2,T}} \le A(T) + B(T)\|u(x,t)\|_{B^3_{2,T}}\left(\|a(t)\|_{C[0,T]+1}\right), \tag{3.12}$$

where $A(T) = A_1(T) + A_2(T)$, $B(T) = B_1(T) + B_2(T)$.

We can prove the following theorem:

Theorem 3.1 *Let 1-4 and*

$$(A(T) + 2)(A(T) + 3)B(T) < 1 \tag{3.13}$$

be fulfilled.

Then problem (2.1)-(2.3), (2.13) *has a unique solution in the ball* $k = k_R\left(\|z\|_{E^3_T} \le R = A(T) + 2\right)$ *of the space* E^3_T.

Proof. In the space E^3_T consider the equation

$$z = \Phi z, \tag{3.14}$$

where $z = \{u, a\}$ are the components $\Phi_i(u,a)\ \ (l = 1,2)$ of the operator $\Phi(u,a)$ determined by the right sides of equations (3.5), (3.7).

Consider the operator $\Phi(u,a)$ in the ball $k = k_R$ from E^3_T. Similar to (3.12) we get that for any $z, z_1, z_2 \in k_R$ the following estimations are valid

$$\|\Phi z\|_{E^3_t} \le A(T) + B(T)\|u(x,l)\|_{B^3_{2,T}}\left(\|a(t)\|_{c[0,T]} + 1\right) \tag{3.15}$$
$$\le A(T) + B(T)(A(t) + 2)(A(T) + 3),$$

$$\|\Phi z_1 - \Phi z_2\|_{E^3_t} \le B(T)(R+1)\left(\|u_1(k,t) - u_2(x,t)\|_{B^3_{2,T}} + \|a_1(t) - a_2(t)\|_{C[0,T]}\right). \tag{3.16}$$

Then allowing for (3.13), from estimations (3.15) and (3.16), it follows that the operator Φ acts in the ball $k = k_R$ and is contractive. Therefore in the ball $k = k_R$ the operator Φ has a unique fixed point $\{u, a\}$, that is unique solution of equation (3.14) in the ball $k = k_R$ i.e. $\{u, a\}$ is a unique solution of system (3.5), (3.7) in the ball $k = k_R$.

The function $u(x,t)$ as an elements of the space $B^3_{2,T}$ is continuous, and has continiuious derivatives $u_x(x,t)$ and $u_{xx}(x,t)$ in D_T.

Now from (3.2) we have:

$$\left(\sum_{k=1}^{\infty}\left(\lambda_k\left\|u_k''(t)\right\|_{C[0,T]}\right)^2\right)^{1/2} \leq \sqrt{2}\left(\sum_{k=1}^{\infty}\left(\lambda_k^3\left\|u_k(t)\right\|_{C[0,T]}\right)^2\right)^{1/2}$$

$$+\sqrt{2}\left\|\|a(t)+f_x(x,l)\|_{C[0,T]}\right\|_{L_2(0,1)}.$$

Hence it follows that $u_{tt}(x,t)$ is continuous D_T.

It is easy to check that equation (2.1) and conditions (2.2), (2.3),(2.13) are satisfied in the ordinary sense. Consequently, $\{u\,(x,t)\,,a\,(t)\}$ is the solution of problem (2.1)-(2.3), (2.13). By lemma 2, this solution is unique. The theorem is proved.

By means of theorem 1 we can prove

Theorem 3.2 *Let all the conditions of theorem 2 and*

$$\varphi(1)=h(0), \quad \psi(1)+\int_0^T M(t)h(t)dt=h'(0),$$

$$\left(\|M(t)\|_{C[0,T]}+\frac{1}{2}(A(T)+2)(A(T)+3)\right)T<1$$

be fulfilled. Then in the ball $k=k_R\left(\|z\|_{E^3_T}\leq R=A(T)+2\right)$ *of the space* E^3_T*, problem* (2.1)-(2.4) *has a unique classical solution.*

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