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Inverse scattering problem for a hyperbolic system of first order equations on a semi-axis on a first approximation

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Abstract. For a hyperbolic system of five equations on a semi-axis, by joint consideration of three problems an inverse scattering problem on a first approximation was solved. The coefficients of the considered system are uniquely determined by the scattering operator on a semi-axis.

Keywords. inverse problem, scattering operator, factorization.

Mathematics Subject Classification (2010): 35J25; 35R30

1 Introduction

Inverse scattering problems for different linear systems of first order hyperbolic equations on the axis and semi-axis were studied in the papers of L.P. Nizhnik [6], L.P. Nizhnik and V.G. Tarasov [7], A.S. Fokas and L.Y. Sung [1], N.Sh. Iskenderov [2], M.I. Ismailov [3] and others.

In this paper we study direct and inverse scattering problems for a system of five hyperbolic equations of first order on a semi-axis in the case when there are three given incident waves.

When there are two incident and three scattering waves, these problems were studied in [4], when there are four incident and one scattering or one incident and four scattering waves, in [2].

2 Scattering problem on a semi-axis.

When there are three incident and two scattering waves, the direct scattering problem was studied in [5] under other boundary conditions.

On a semi-axis $x \geq 0$ consider a system of equations of the form:

$$\xi_i \frac{\partial U_i(x,t)}{\partial t} - \frac{\partial U_i(x,t)}{\partial x} = \sum_{j=1}^{5} C_{ij}(x,t) U_j(x,t), \tag{2.1}$$

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where $C_{ij}(x,t)$are complex-valued functions measurable with respect to x and t satisfying the conditions:

$$|C_{ij}(x,t)| \leq C\left[(1+|x|)(1+|t|)\right]^{-1-\varepsilon}, \quad c>0, \varepsilon>0, \tag{2.2}$$

moreover

$$C_{ii}(x,t)=0, \ i=\overline{1,5}; \ \xi_1>\xi_2>\xi_3>0>\xi_4>\xi_5, \ -\infty<t<\infty.$$

Let us consider system (2.1) on a semi-axis under three different boundary conditions:

$$U_4^1(0,t)=U_3^1(0,t),$$
$$U_5^1(0,t)=U_1^1(0,t), \tag{2.3}$$
$$U_4^2(0,t)=U_1^2(0,t),$$
$$U_5^2(0,t)=U_2^2(0,t), \tag{2.4}$$
$$U_4^3(0,t)=U_2^3(0,t),$$
$$U_5^3(0,t)=U_3^3(0,t). \tag{2.5}$$

Any essentially bounded solution $U(x,t)=\{U_1(x,t),U_2(x,t),...,U_5(x,t)\}$ of the system (2.1) with the coefficients $C_{ij}(x,t), \ i.j=\overline{1,5}$, satisfying condition (2.2) admits on the semi-axis $x\geq 0$ the following asymptotic representations:

$$U_j(x,t)=a_j\left(t+\xi_j x\right)+o(1), \ \ j=1,2,3,$$
$$U_j(x,t)=b_j\left(t+\xi_j x\right)+o(1), \ \ j=4,5, x\to+\infty, \tag{2.6}$$

where $a_j\left(s\right)\in L_\infty(-\infty,+\infty), \ \ j=1,2,3$, determine the incident waves, while $b_j\left(s\right)\in L_\infty(-\infty,+\infty)$, $j=4,5$ the scattering ones.

The scattering problem for system (2.1) is in finding the solution to the system (2.1) by the given incidence waves and boundary conditions for $x=0$.

The scattering problem under joint consideration of the first, second and third problems is stated as follows: by the given function $a_1\left(s\right), a_2\left(s\right), a_3\left(s\right)\in L_\infty(-\infty,+\infty)$ find the solution

$$U^k(x,t)\in L_\infty\left((0,+\infty)\times(-\infty,+\infty),C^3\right), \ (k=1,2,3)$$

of the first, second and third problems for which in L_∞ the following asymptotic representations are valid:

$$U_i^k(x,t)=a_i(t+\xi_i\cdot x)+o(1), \ x\to+\infty\,, \ i,k=1,2,3;$$

where $U^k(x,t)=\left\{U_1^k(x,t)....U_5^k(x,t)\right\}$ ..

Theorem 1. Let the coefficients $C_{ij}(x,t), \ \ i.j=\overline{1,5}$, of the system (2.1), satisfy conditions (2.2). Then there exists a unique solution of the scattering problem on the semi-axis $x\geq 0$ for the system (2.1) with arbitrary given incident waves

$$a_i\left(s\right)\in L_\infty(-\infty,+\infty), \ i=1,2,3.$$

The proof is this theorem is similar to one in [5].

Note that the scattering problem for the k-th $(k=1,2,3)$ problem is equivalent to the following system of integral equations:

$$U_i^k(x,t)=a_i(t+\xi_i x)+\int_x^{+\infty}\sum_{j=1}^{5}C_{ij}(y,t+\xi_i(x-y))U_j^k(y,t+\xi_i(x-y))dy, \ i=1,2,3,$$

$$U_i^k(x,t) = b_i^k(t+\xi_i x)$$
$$+\int_x^{+\infty}\sum_{j=1}^{5} C_{ij}(y,t+\xi_i(x-y))U_j^k(y,t+\xi_i(x-y))dy,\ i=4,5, \tag{2.7}$$

where the functions $b_4^k(s), b_5^k(s), k=1,2,3$, are expressed by $a_1(s), a_2(s), a_3(s)$ the coefficients
$C_{ij}(x,t),\ \left(i.j=\overline{1,5}\right)$ and the solutions of the first, second and third problems, respectively, in the following way:

$$\begin{cases} b_4^1(t) = a_3(t) \\ +\int\limits_0^{+\infty}\sum\limits_{j=1}^{5}\Big[C_{3j}(y,t-\xi_3 y)U_j^1(y,t-\xi_3 y) - C_{4j}(y,t-\xi_4 y)U_j^1(y,t-\xi_4 y)\Big]dy, \\ b_5^1(t) = a_1(t) \\ +\int\limits_0^{+\infty}\sum\limits_{j=1}^{5}\Big[C_{1j}(y,t-\xi_1 y)U_j^1(y,t-\xi_1 y) - C_{5j}(y,t-\xi_5 y)U_j^1(y,t-\xi_5 y)\Big]dy; \end{cases} \tag{2.8}$$

$$\begin{cases} b_4^2(t) = a_1(t) \\ +\int\limits_0^{+\infty}\sum\limits_{j=1}^{5}\Big[C_{1j}(y,t-\xi_1 y)U_j^2(y,t-\xi_1 y) - C_{4j}(y,t-\xi_4 y)U_j^2(y,t-\xi_4 y)\Big]dy, \\ b_5^2(t) = a_2(t) \\ +\int\limits_0^{+\infty}\sum\limits_{j=1}^{5}\Big[C_{2j}(y,t-\xi_2 y)U_j^1(y,t-\xi_2 y) - C_{5j}(y,t-\xi_5 y)U_j^1(y,t-\xi_5 y)\Big]dy; \end{cases} \tag{2.9}$$

$$\begin{cases} b_4^3(t) = a_2(t) \\ +\int\limits_0^{+\infty}\sum\limits_{j=1}^{5}\Big[C_{2j}(y,t-\xi_2 y)U_j^3(y,t-\xi_2 y) - C_{4j}(y,t-\xi_4 y)U_j^1(y,t-\xi_4 y)\Big]dy, \\ b_5^3(t) = a_3(t) \\ +\int\limits_0^{+\infty}\sum\limits_{j=1}^{5}\Big[C_{3j}(y,t-\xi_3 y)U_j^3(y,t-\xi_3 y) - C_{5j}(y,t-\xi_5 y)U_j^1(y,t-\xi_5 y)\Big]dy. \end{cases} \tag{2.10}$$

It follows from theorem (2.1) that to each vector-function $a(s) = (a_1(s), a_2(s), a_3(s)) \in L_\infty(R)$ giving the incident waves there correspond the solutions of three scattering problems of the system (2.1) with boundary conditions (2.3), (2.4), (2.5) and the given asymptotics

$$U_4^k(x,t) = b_4^k\,(t+\xi_4 x) + o(1),\ \ x\to+\infty,$$
$$U_5^k(x,t) = b_5^k\,(t+\xi_5 x) + o(1),\ \ x\to+\infty, \tag{2.11}$$

i.e. the vector of scattering waves $b(t) = \left(b^1(t), b^2(t), b^3(t)\right)$, where $b^k(t) = \left(b_4^k(t), b_5^k(t)\right)$, $k=1,2,3$. Relation (2.11) follows from (2.7) and conditions (2.2).

Thus, in the space of essentially bounded functions, we determined the operator S that takes $a(t),$ to $b(t)$:

$$S\begin{pmatrix} a_1(t) \\ a_2(t) \\ a_3(t) \end{pmatrix} = \left(b_4^1(t), b_5^1(t), b_4^2(t), b_5^2(t), b_4^3(t), b_5^3(t)\right), \tag{2.12}$$

T denotes the transposition sign.
Here,

$$S = \left(S^1, S^{2,}, S^3\right),\ \text{a } S^k = \begin{pmatrix} S_{11}^k & S_{12}^k & S_{13}^k \\ S_{21}^k & S_{22}^k & S_{23}^k \end{pmatrix},\ k=1,2,3.$$

$$b_4^k(t) = S_{11}^k a_1(t) + S_{12}^k a_2(t) + S_{13}^k a_3(t),$$

$$b_5^k(t) = S_{21}^k a_1(t) + S_{22}^k a_2(t) + S_{23}^k a_3(t), \quad k = 1,2,3. \tag{2.13}$$

From (2.8), (2.9), (2.10) it follows that the elements of the operator $S^k (k = 1, 2, 3)$ have the form:

$$S_{11}^1 = F_{11}^1,\ S_{12}^1 = F_{12}^1,\ S_{13}^1 = I + F_{13}^1,$$

$$S_{21}^1 = I + F_{21}^1,\ S_{22}^1 = F_{22}^1,\ S_{23}^1 = F_{23}^1,$$

$$S_{11}^2 = I + F_{11}^2,\ S_{12}^2 = F_{12}^2,\ S_{13}^2 = F_{13}^2, S_{21}^2 = F_{21}^2,$$

$$S_{22}^2 = I + F_{22}^2,\ S_{23}^2 = F_{23}^2,\ S_{11}^3 = F_{11}^3, S_{12}^3 = I + F_{12}^3,$$

$$S_{13}^3 = F_{13}^3,\ S_{21}^3 = F_{21}^3,\ S_{22}^3 = F_{22}^3, S_{23}^3 = I + F_{23}^3 \tag{2.14}$$

where the operators F_{ij}^k, $(k, j = 1, 2, 3; i = 1, 2)$ are Fredholm integral operators.

3 The inverse scattering problem on a semi-axis on a first approximation

The inverse problem for the system (2.1) is in finding the coefficients of the system (2.1) by the given scattering operator S on a semi-axis.

Here the coefficients of the system (2.1) are restored by the scattering operator on a semi-axis constructed on a first approximation. It is constructed in the explicit form.

For the first problem as a zero order approximation we take

$$U_1^{(1,0)}(x,t) = a_1(t + \xi_1 x),$$

$$U_2^{(1,0)}(x,t) = a_2(t + \xi_2 x),$$

$$U_3^{(1,0)}(x,t) = a_3(t + \xi_3 x),$$

$$U_4^{(1,0)}(x,t) = a_3(t + \xi_4 x),$$

$$U_5^{(1,0)}(x,t) = a_1(t + \xi_5 x);$$

for the second problem

$$U_1^{(2,0)}(x,t) = a_1(t + \xi_1 x),$$

$$U_2^{(2,0)}(x,t) = a_2(t + \xi_2 x),$$

$$U_3^{(2,0)}(x,t) = a_3(t + \xi_3 x),$$

$$U_4^{(2,0)}(x,t) = a_1(t + \xi_4 x),$$

$$U_5^{(2,0)}(x,t) = a_2(t + \xi_5 x);$$

for the third one

$$U_1^{(3,0)}(x,t) = a_1(t + \xi_1 x),$$

$$U_2^{(3,0)}(x,t) = a_2(t + \xi_2 x),$$

$$U_3^{(3,0)}(x,t) = a_3(t + \xi_3 x),$$

$$U_4^{(3,0)}(x,t) = a_2(t + \xi_4 x),$$

$$U_5^{(3,0)}(x,t) = a_3(t + \xi_5 x).$$

Then in equalities (2.8), (2.9) and (2.10) the first order approximations will be:

$$\begin{cases} b_4^1(t) = a_3(t) + \int\limits_0^{+\infty} \sum\limits_{j=1}^{5} \Big[C_{3j}(y,t-\xi_3 y) U_j^{(1,0)}(y,t-\xi_3 y) \\ \qquad\qquad -C_{4j}(y,t-\xi_4 y) U_j^{(1,0)}(y,t-\xi_4 y) \Big] \, dy, \\ b_5^1(t) = a_1(t) + \int\limits_0^{+\infty} \sum\limits_{j=1}^{5} \Big[C_{1j}(y,t-\xi_1 y) U_j^{(1,0)}(y,t-\xi_1 y) \\ \qquad\qquad -C_{5j}(y,t-\xi_5 y) U_j^{(1,0)}(y,t-\xi_5 y) \Big] \, dy; \end{cases} \tag{3.1}$$

$$\begin{cases} b_4^2(t) = a_3(t) + \int\limits_0^{+\infty} \sum\limits_{j=1}^{5} \Big[C_{1j}(y,t-\xi_1 y) U_j^{(2,0)}(y,t-\xi_1 y) \\ \qquad\qquad -C_{4j}(y,t-\xi_4 y) U_j^{(2,0)}(y,t-\xi_4 y) \Big] \, dy, \\ b_5^2(t) = a_2(t) + \int\limits_0^{+\infty} \sum\limits_{j=1}^{5} \Big[C_{2j}(y,t-\xi_2 y) U_j^{(2,0)}(y,t-\xi_2 y) \\ \qquad\qquad -C_{5j}(y,t-\xi_5 y) U_j^{(2,0)}(y,t-\xi_5 y) \Big] \, dy; \end{cases} \tag{3.2}$$

$$\begin{cases} b_4^3(t) = a_2(t) + \int\limits_0^{+\infty} \sum\limits_{j=1}^{5} \Big[C_{2j}(y,t-\xi_2 y) U_j^{(3,0)}(y,t-\xi_2 y) \\ \qquad\qquad -C_{4j}(y,t-\xi_4 y) U_j^{(3,0)}(y,t-\xi_4 y) \Big] \, dy, \\ b_5^3(t) = a_3(t) + \int\limits_0^{+\infty} \sum\limits_{j=1}^{5} \Big[C_{3j}(y,t-\xi_1 y) U_j^{(3,0)}(y,t-\xi_1 y) \\ \qquad\qquad -C_{5j}(y,t-\xi_5 y) U_j^{(3,0)}(y,t-\xi_5 y) \Big] \, dy. \end{cases} \tag{3.3}$$

respectively.

Taking into account in (2.13) and (2.14), we have:

$$F_{11}^1(t,\tau) = \begin{cases} \frac{1}{\xi_3-\xi_5} C_{35}\left(\frac{\tau-t}{\xi_5-\xi_3}, \frac{\xi_5 t-\xi_3\tau}{\xi_5-\xi_3}\right) \\ \qquad + \frac{1}{\xi_4-\xi_5} C_{45}\left(\frac{\tau-t}{\xi_5-\xi_4}, \frac{\xi_5 t-\xi_4\tau}{\xi_5-\xi_4}\right), \quad -\infty < \tau < t, \\ \frac{1}{\xi_1-\xi_5} C_{31}\left(\frac{\tau-t}{\xi_1-\xi_3}, \frac{\xi_1 t-\xi_3\tau}{\xi_1-\xi_3}\right) \\ \qquad + \frac{1}{\xi_1-\xi_4} C_{41}\left(\frac{\tau-t}{\xi_1-\xi_4}, \frac{\xi_1 t-\xi_4\tau}{\xi_1-\xi_4}\right), \quad t < \tau < +\infty; \end{cases} \tag{3.4}$$

$$F_{12}^1(t,\tau) = \begin{cases} 0 \qquad\qquad -\infty < \tau < t \\ \frac{1}{\xi_2-\xi_5} C_{32}\left(\frac{\tau-t}{\xi_2-\xi_3}, \frac{\xi_2 t-\xi_3\tau}{\xi_2-\xi_3}\right) \\ \qquad + \frac{1}{\xi_2-\xi_4} C_{42}\left(\frac{\tau-t}{\xi_2-\xi_4}, \frac{\xi_2 t-\xi_4\tau}{\xi_2-\xi_4}\right), \quad t < \tau < +\infty \end{cases} \tag{3.5}$$

$$F_{13}^1(t,\tau) = \begin{cases} \frac{1}{\xi_3-\xi_4} C_{34}\left(\frac{\tau-t}{\xi_4-\xi_3}, \frac{\xi_4 t-\xi_3\tau}{\xi_4-\xi_3}\right), \quad -\infty < \tau < t, \\ \frac{1}{\xi_3-\xi_4} C_{43}\left(\frac{\tau-t}{\xi_3-\xi_4}, \frac{\xi_3 t-\xi_4\tau}{\xi_3-\xi_4}\right), \quad t < \tau < +\infty \end{cases} \tag{3.6}$$

$$F_{23}^1(t,\tau) = \begin{cases} \frac{1}{\xi_1-\xi_5} C_{15}\left(\frac{\tau-t}{\xi_4-\xi_3}, \frac{\xi_5 t-\xi_1\tau}{\xi_5-\xi_1}\right), \quad -\infty < \tau < t, \\ \frac{1}{\xi_1-\xi_5} C_{51}\left(\frac{\tau-t}{\xi_1-\xi_5}, \frac{\xi_1 t-\xi_5\tau}{\xi_1-\xi_5}\right), \quad t < \tau < +\infty \end{cases} \tag{3.7}$$

$$F_{22}^1(t,\tau)=\begin{cases}\frac{1}{\xi_1-\xi_2}C_{12}\left(\frac{\tau-t}{\xi_2-\xi_1},\frac{\xi_2 t-\xi_1\tau}{\xi_2-\xi_1}\right), & -\infty<\tau<t,\\ \frac{1}{\xi_2-\xi_5}C_{52}\left(\frac{\tau-t}{\xi_2-\xi_5},\frac{\xi_2 t-\xi_5\tau}{\xi_2-\xi_5}\right), & t<\tau<+\infty\end{cases}\tag{3.8}$$

$$F_{23}^1(t,\tau)=\begin{cases}\frac{1}{\xi_1-\xi_3}C_{13}\left(\frac{\tau-t}{\xi_3-\xi_1},\frac{\xi_3 t-\xi_1\tau}{\xi_3-\xi_1}\right)\\ \qquad+\frac{1}{\xi_1-\xi_4}C_{45}\left(\frac{\tau-t}{\xi_4-\xi_1},\frac{\xi_4 t-\xi_1\tau}{\xi_4-\xi_1}\right), & -\infty<\tau<t\\ \frac{1}{\xi_3-\xi_5}C_{53}\left(\frac{\tau-t}{\xi_3-\xi_5},\frac{\xi_3 t-\xi_5\tau}{\xi_3-\xi_5}\right)\\ \qquad+\frac{1}{\xi_4-\xi_5}C_{54}\left(\frac{\tau-t}{\xi_4-\xi_5},\frac{\xi_4 t-\xi_5\tau}{\xi_4-\xi_5}\right), & t<\tau<+\infty\end{cases}\tag{3.9}$$

$$F_{11}^2(t,\tau)=\begin{cases}\frac{1}{\xi_1-\xi_4}C_{14}\left(\frac{\tau-t}{\xi_4-\xi_1},\frac{\xi_4 t-\xi_1\tau}{\xi_4-\xi_1}\right), & -\infty<\tau<t\\ \frac{1}{\xi_4-\xi_1}C_{41}\left(\frac{\tau-t}{\xi_1-\xi_4},\frac{\xi_1 t-\xi_4\tau}{\xi_1-\xi_4}\right), & t<\tau<+\infty\end{cases}\tag{3.10}$$

$$F_{12}^2(t,\tau)=\begin{cases}\frac{1}{\xi_1-\xi_2}C_{12}\left(\frac{\tau-t}{\xi_2-\xi_1},\frac{\xi_2 t-\xi_1\tau}{\xi_2-\xi_1}\right)\\ \qquad+\frac{1}{\xi_1-\xi_5}C_{15}\left(\frac{\tau-t}{\xi_5-\xi_4},\frac{\xi_5 t-\xi_1\tau}{\xi_5-\xi_4}\right), & -\infty<\tau<t\\ \frac{1}{\xi_4-\xi_2}C_{42}\left(\frac{\tau-t}{\xi_2-\xi_4},\frac{\xi_2 t-\xi_4\tau}{\xi_2-\xi_4}\right)\\ \qquad-\frac{1}{\xi_4-\xi_5}C_{45}\left(\frac{\tau-t}{\xi_5-\xi_4},\frac{\xi_5 t-\xi_4\tau}{\xi_5-\xi_4}\right), & t<\tau<+\infty\end{cases}\tag{3.11}$$

$$F_{13}^2(t,\tau)=\begin{cases}\frac{1}{\xi_1-\xi_3}C_{13}\left(\frac{\tau-t}{\xi_3-\xi_1},\frac{\xi_3 t-\xi_1\tau}{\xi_3-\xi_1}\right), & -\infty<\tau<t\\ \frac{1}{\xi_3-\xi_4}C_{43}\left(\frac{\tau-t}{\xi_3-\xi_4},\frac{\xi_3 t-\xi_4\tau}{\xi_3-\xi_4}\right), & t<\tau<+\infty\end{cases}\tag{3.12}$$

$$F_{21}^2(t,\tau)=\begin{cases}\frac{1}{\xi_2-\xi_4}C_{24}\left(\frac{\tau-t}{\xi_4-\xi_2},\frac{\xi_4 t-\xi_2\tau}{\xi_4-\xi_2}\right), & -\infty<\tau<t,\\ \frac{1}{\xi_1-\xi_2}C_{21}\left(\frac{\tau-t}{\xi_1-\xi_2},\frac{\xi_1 t-\xi_2\tau}{\xi_1-\xi_2}\right)+\frac{1}{\xi_1-\xi_5}C_{51}\left(\frac{\tau-t}{\xi_1-\xi_5},\frac{\xi_1 t-\xi_5\tau}{\xi_1-\xi_5}\right)-\\ -\frac{1}{\xi_4-\xi_5}C_{54}\left(\frac{\tau-t}{\xi_4-\xi_5},\frac{\xi_4 t-\xi_5\tau}{\xi_4-\xi_5}\right), & t<\tau<+\infty;\end{cases}\tag{3.13}$$

$$F_{22}^2(t,\tau)=\begin{cases}\frac{1}{\xi_2-\xi_5}C_{25}\left(\frac{\tau-t}{\xi_5-\xi_2},\frac{\xi_2 t-\xi_1\tau}{\xi_5-\xi_2}\right), & -\infty<\tau<t,\\ \frac{1}{\xi_5-\xi_2}C_{52}\left(\frac{\tau-t}{\xi_2-\xi_5},\frac{\xi_2 t-\xi_5\tau}{\xi_2-\xi_5}\right), & t<\tau<+\infty\end{cases}\tag{3.14}$$

$$F_{23}^2(t,\tau)=\begin{cases}\frac{1}{\xi_2-\xi_5}C_{23}\left(\frac{\tau-t}{\xi_3-\xi_2},\frac{\xi_3 t-\xi_2\tau}{\xi_3-\xi_2}\right), & -\infty<\tau<t,\\ \frac{1}{\xi_5-\xi_3}C_{53}\left(\frac{\tau-t}{\xi_3-\xi_5},\frac{\xi_3 t-\xi_5\tau}{\xi_3-\xi_5}\right), & t<\tau<+\infty\end{cases}\tag{3.15}$$

$$F_{11}^3(t,\tau)=\begin{cases}0, & -\infty<\tau<t\\ \frac{1}{\xi_1-\xi_2}C_{21}\left(\frac{\tau-t}{\xi_1-\xi_2},\frac{\xi_1 t-\xi_2\tau}{\xi_1-\xi_2}\right)\\ \qquad+\frac{1}{\xi_1-\xi_4}C_{41}\left(\frac{\tau-t}{\xi_1-\xi_4},\frac{\xi_1 t-\xi_4\tau}{\xi_1-\xi_4}\right), & t<\tau<+\infty\end{cases}\tag{3.16}$$

$$F_{12}^3(t,\tau)=\begin{cases}\frac{1}{\xi_2-\xi_4}C_{24}\left(\frac{\tau-t}{\xi_4-\xi_2},\frac{\xi_4 t-\xi_2\tau}{\xi_4-\xi_2}\right), & -\infty<\tau<t,\\ \frac{1}{\xi_2-\xi_4}C_{42}\left(\frac{\tau-t}{\xi_2-\xi_4},\frac{\xi_2 t-\xi_4\tau}{\xi_2-\xi_4}\right), & t<\tau<+\infty\end{cases}\tag{3.17}$$

$$F_{13}^2(t,\tau)=\begin{cases}\frac{1}{\xi_2-\xi_4}C_{23}\left(\frac{\tau-t}{\xi_3-\xi_2},\frac{\xi_3 t-\xi_2\tau}{\xi_3-\xi_2}\right)+\frac{1}{\xi_2-\xi_5}C_{25}\left(\frac{\tau-t}{\xi_5-\xi_2},\frac{\xi_5 t-\xi_2\tau}{\xi_5-\xi_2}\right)+\\ +\frac{1}{\xi_3-\xi_4}C_{43}\left(\frac{\tau-t}{\xi_3-\xi_4},\frac{\xi_3 t-\xi_4\tau}{\xi_3-\xi_4}\right)\\ \qquad+\frac{1}{\xi_4-\xi_5}C_{45}\left(\frac{\tau-t}{\xi_5-\xi_4},\frac{\xi_5 t-\xi_4\tau}{\xi_5-\xi_4}\right), & -\infty<\tau<t,\\ 0, & t<\tau<+\infty\end{cases}\tag{3.18}$$

$$F^3_{21}(t,\tau)=\begin{cases}0, & -\infty<\tau<t\\ \frac{1}{\xi_1-\xi_3}C_{31}\left(\frac{\tau-t}{\xi_1-\xi_3},\frac{\xi_1 t-\xi_3\tau}{\xi_1-\xi_3}\right) & \\ \qquad+\frac{1}{\xi_1-\xi_5}C_{51}\left(\frac{\tau-t}{\xi_1-\xi_5},\frac{\xi_1 t-\xi_5\tau}{\xi_1-\xi_5}\right), & t<\tau<+\infty\end{cases} \tag{3.19}$$

$$F^3_{22}(t,\tau)=\begin{cases}\frac{1}{\xi_3-\xi_4}C_{34}\left(\frac{\tau-t}{\xi_4-\xi_3},\frac{\xi_4 t-\xi_2\tau}{\xi_4-\xi_2}\right), & -\infty<\tau<t,\\ \frac{1}{\xi_2-\xi_3}C_{32}\left(\frac{\tau-t}{\xi_2-\xi_3},\frac{\xi_2 t-\xi_3\tau}{\xi_2-\xi_3}\right)+\frac{1}{\xi_2-\xi_5}C_{52}\left(\frac{\tau-t}{\xi_2-\xi_5},\frac{\xi_2 t-\xi_5\tau}{\xi_2-\xi_5}\right)+ & \\ \qquad+\frac{1}{\xi_4-\xi_5}C_{54}\left(\frac{\tau-t}{\xi_4-\xi_5},\frac{\xi_4 t-\xi_5\tau}{\xi_4-\xi_5}\right), & t<\tau<+\infty\end{cases} \tag{3.20}$$

$$F^3_{23}(t,\tau)=\begin{cases}\frac{1}{\xi_3-\xi_5}C_{35}\left(\frac{\tau-t}{\xi_5-\xi_3},\frac{\xi_5 t-\xi_3\tau}{\xi_5-\xi_3}\right), & -\infty<\tau<t,\\ \frac{1}{\xi_3-\xi_5}C_{53}\left(\frac{\tau-t}{\xi_3-\xi_5},\frac{\xi_3 t-\xi_5\tau}{\xi_3-\xi_5}\right), & t<\tau<+\infty,\end{cases} \tag{3.21}$$

where $F^{1,2,3}_{ij}(t,\tau)$ are the kernels of the operators $F_{jk}(k=1,2,3)$. It is easy to see that the coefficients of the system (2.1) are found by the operator S in the following way:

$$C_{12}(x,t)=(\xi_1-\xi_2)F^1_{22}(\xi_1 x+t,\xi_2 x+t),$$
$$C_{13}(x,t)=(\xi_1-\xi_3)F^2_{13}(\xi_1 x+t,\xi_3 x+t),$$
$$C_{14}(x,t)=(\xi_1-\xi_4)F^2_{11}(\xi_1 x+t,\xi_4 x+t),$$
$$C_{15}(x,t)=(\xi_1-\xi_5)F^1_{21}(\xi_1 x+t,\xi_5 x+t),$$
$$C_{23}(x,t)=(\xi_2-\xi_3)F^2_{23}(\xi_2 x+t,\xi_3 x+t),$$
$$C_{24}(x,t)=(\xi_2-\xi_4)F^2_{21}(\xi_2 x+t,\xi_4 x+t)=(\xi_2-\xi_4)F^3_{12}(\xi_2 x+t,\xi_4 x+t),$$
$$C_{25}(x,t)=(\xi_2-\xi_5)F^2_{22}(\xi_2 x+t,\xi_5 x+t),$$
$$C_{34}(x,t)=(\xi_3-\xi_4)F^3_{22}(\xi_3 x+t,\xi_4 x+t)=(\xi_3-\xi_4)F^1_{13}(\xi_3 x+t,\xi_4 x+t),$$
$$C_{35}(x,t)=(\xi_3-\xi_5)F^3_{23}(\xi_3 x+t,\xi_5 x+t),$$
$$C_{41}(x,t)=(\xi_4-\xi_1)F^2_{11}(\xi_4 x+t,\xi_1 x+t),$$
$$C_{42}(x,t)=(\xi_2-\xi_4)F^3_{12}(\xi_4 x+t,\xi_2 x+t),$$
$$C_{43}(x,t)=(\xi_3-\xi_4)F^2_{13}(\xi_4 x+t,\xi_3 x+t),$$
$$C_{51}(x,t)=(\xi_1-\xi_5)F^1_{21}(\xi_5 x+t,\xi_1 x+t),$$
$$C_{52}(x,t)=(\xi_2-\xi_5)F^1_{22}(\xi_5 x+t,\xi_2 x+t)=(\xi_5-\xi_2)F^2_{22}(\xi_5 x+t,\xi_2 x+t),$$
$$C_{53}(x,t)=(\xi_3-\xi_5)F^3_{23}(\xi_5 x+t,\xi_3 x+t),$$
$$C_{32}(x,t)=(\xi_2-\xi_3)\left[F^1_{12}(\xi_3 x+t,\xi_2 x+t)-F^3_{12}(\xi_3 x+t,\xi_2 x+t)\right],$$
$$C_{31}(x,t)=(\xi_1-\xi_3)\left[F^3_{21}(\xi_3 x+t,\xi_1 x+t)-F^1_{21}(\xi_3 x+t,\xi_1 x+t)\right],$$
$$C_{21}(x,t)=(\xi_1-\xi_2)\left[F^3_{11}(\xi_2 x+t,\xi_1 x+t)-F^2_{11}(\xi_2 x+t,\xi_1 x+t)\right],$$
$$C_{54}(x,t)=(\xi_4-\xi_5)\left[F^1_{23}(\xi_5 x+t,\xi_4 x+t)-F^3_{23}(\xi_5 x+t,\xi_4 x+t)\right],$$
$$C_{45}(x,t)=(\xi_5-\xi_4)\left[F^2_{12}(\xi_4 x+t,\xi_5 x+t)+F^3_{12}(\xi_4 x+t,\xi_5 x+t)\right]. \tag{3.22}$$

Note that from the remaining relations (3.4)-(3.21) we have:

$$F^2_{21}(t,\tau)=F^3_{12}(t,\tau),\ \tau<t;\ \ F^1_{22}(t,\tau)=F^2_{22}(t,\tau),\ \tau>t;$$

$$F^1_{13}(t,\tau) = F^3_{22}(t,\tau),\ \tau < t;\ \ F^1_{23}(t,\tau) = F^2_{11}(t,\tau) + F^2_{13}(t,\tau),\ \tau < t;$$
$$F^2_{12}(t,\tau) = F^1_{22}(t,\tau) + F^1_{21}(t,\tau),\ \ \tau < t;$$
$$F^2_{21}(t,\tau) = F^3_{11}(t,\tau) - F^2_{11}(t,\tau) - F^1_{22}(t,\tau) - F^1_{23}(t,\tau) + F^3_{23}(t,\tau),\ \tau > t;$$
$$F^2_{13}(t,\tau) = F^2_{23}(t,\tau) + F^2_{22}(t,\tau) + F^2_{13}(t,\tau) - F^2_{12}(t,\tau) - F^3_{12}(t,\tau),\ \tau < t;$$
$$F^3_{22}(t,\tau) = F^1_{12}(t,\tau) - F^3_{12}(t,\tau) + F^1_{22}(t,\tau) + F^1_{23}(t,\tau) - F^3_{23}(t,\tau),\ \tau > t; \qquad (3.23)$$

The scattering operator S has 18 elements on the axis or 36 elements on the semi-axis. From 36 elements by formula (3.22) we find 20 coefficients of the system (2.1), 16 unnecessary elements are connected with 8 relations on the axis (16 on the semi-axis with respect to t) by formula (3.23).

It is also seen from relations (3.4)-(3.22) that for the system (2.1) three problems should be necessary stated.

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