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Nonexistence of global solutions of the Cauchy problem for the systems of three semilinear hyperbolic equations with positive initial energy

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Abstract. *In this paper we study the Cauchy problem for the system of three semilinear hyperbolic equations . We prove a theorem on the nonexistence of global solutions with positive initial energy.*

Keywords. Semilinear, system, hyperbolic, non-existence, blow-up.

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1 Introduction

We consider the following initial boundary value problem:

$$\begin{cases} u_{1tt} - \Delta u_1 + u_1 + \gamma u_{1t} = f_1(u_1, u_2, u_3), \\ u_{2tt} - \Delta u_2 + u_2 + \gamma u_{2t} = f_2(u_1, u_2, u_3), \\ u_{3tt} - \Delta u_3 + u_3 + \gamma u_{3t} = f_3(u_1, u_2, u_3), \end{cases} \tag{1.1}$$

with initial conditions

$$u_i(0, x) = u_{i0}(x), \quad u_{it}(0, x) = u_{i1}(x), \quad x \in R^n, \quad i = 1, 2, 3, \tag{1.2}$$

where

$$f_1(u_1, u_2, u_3) = |u_1|^{p_1-1} |u_2|^{p_2+1} |u_3|^{p_3+1} u_1,$$
$$f_2(u_1, u_2, u_3) = |u_1|^{p_1+1} |u_2|^{p_2-1} |u_3|^{p_3+1} u_2,$$
$$f_3(u_1, u_2, u_3) = |u_1|^{p_1+1} |u_2|^{p_2+1} |u_3|^{p_3-1} u_3,$$

(u_1, u_2, u_3) are real functions depending on $t \in R_+$ and $x \in R^n$, p_1, p_2, p_3 are real numbers. The goal of this paper is to investigate nonexistence of global solutions to the problem (1.1), (1.2).

There are also numerous results on the nonexistence of the global solution for Cauchy problem of the single Klein–Gordon equation. As far as we know the first result about blow up of solutions for a strongly damped equation which is the result of H. Levine [7], where by using the concavity method introduced in [8] he got sufficient conditions for blow up of

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solutions to the Cauchy problem for a nonlinear differential-operator equation in a Hilbert space.

The nonexistence of global solutions of nonlinear wave equations with positive initial energy was considered in [14]. Later various researches were conducted in this direction among which we mention the article [4], [6], [9]-[11], [14]-[16].We give some of the results obtained in this direction. The nonexistence of global solutions of a generalized fourth-order Klein–Gordon equation with positive initial energy was analyzed in [6]. In the paper [4], [5], [12] the authors obtained the sufficient conditions on initial functions for which the initial boundary value problem for second order quasilinear strongly damped wave equations blows up in a finite time. A fairly comprehensive picture of the studies in this direction can be gained from the monograph [13].

A mixed problem for systems of two semilinear wave equations with viscosity and with memory was studied in [12], where the nonexistence of global solutions with positive initial energy was proved. The nonexistence of global solutions of the problem

$$\begin{cases} u_{1tt} - \Delta u_1 + u_1 + \gamma u_{1t} = g_1(u_1, u_2) \\ u_{2tt} - \Delta u_2 + u_i + \gamma u_{2t} = g_2(u_1, u_2), \end{cases} \tag{1.3}$$

$$u_i(0,x) = u_{i0}(x), \quad u_{it}(0,x) = u_{i1}(x), \quad x \in R^n, \quad i = 1,2, \tag{1.4}$$

where

$$g_1(u_1,u_2) = |u_1|^{p-1}|u_2|^{p+1}u_1, fg(u_1,u_2) = |u_1|^{p+1}|u_2|^{p-1}u_2,$$

with negative initial energy was studied in [17], [18]. In the case when

$$g_1(u_1,u_2) = |u_1|^{p-1}|u_2|^{q+1}u_1, g_2(u_1,u_2) = |u_1|^{p+1}|u_2|^{q-1}u_2$$

the non-existence of global solutions for the problem (1.3), (1.4) was investigated in [2].

The absence of global solutions with positive initial energy for systems of semilinear hyperbolic equations

$$u_{itt} - \Delta u_i + u_i + \gamma u_{it} = \sum_{\substack{i,j=1 \\ i\neq j}}^{m} |u_j|^{p_j}|u_i|^{p_i}u_i\,, \quad i = 1,2,...,m$$

was investigated in [1].

2 Formulation of the problem and main results

In what follows, we denote the norm on the space $L_2(R^n)$ by $|\cdot|$, the inner product on $L_2(R^n)$ by $\langle\cdot,\cdot\rangle$, and the norm on the Sobolev space $H^1 = W_2^1(R^n)$ by $\|u\| = \left[|\nabla u|^2 + |u|^2\right]$,where ∇ is the gradient. The constants C and c used throughout this paper are positive generic constants that may be different in various occurrences.

Before introducing our results, we shall impose the following assumptions.

Assume that

$$n \geq 2, \; p_j \geq 0, \; j = 1,2,3, \tag{2.1}$$

and additionally

$$p_1 + p_2 + p_3 \leq \frac{2}{n-2} \;\text{ if } n \geq 3 \tag{2.2}$$

At first we give some auxiliary notations.

In the Hilbert space $\mathcal{H} = L_2(R^n) \times L_2(R^n) \times L_2(R^n)$ we write problem (2.1), (2.2), as the Cauchy problem

$$w'' + Bw' + Aw = F(w), \tag{2.3}$$

$$w(0) = w_0, \ w'(0) = w_1, \tag{2.4}$$

where

$$w' = \begin{bmatrix} u_1' \\ u_2' \\ u_3' \end{bmatrix}, \quad w'' = \begin{bmatrix} u_1'' \\ u_2'' \\ u_3'' \end{bmatrix}, \quad w_0 = \begin{bmatrix} u_{10}(x) \\ u_{20}(x) \\ u_{30}(x) \end{bmatrix}, \ w_1 = \begin{bmatrix} u_{11}(x) \\ u_{21}(x) \\ u_{31}(x) \end{bmatrix}.$$

A and B are linear operators in H defined by the equalities

$$A = \begin{pmatrix} -\Delta + 1 & 0 & 0 \\ 0 & -\Delta + 1 & 0 \\ 0 & 0 & -\Delta + 1 \end{pmatrix}, \qquad D(A) = H^2 \times H^2 \times H^2, \ H^2 = W_2^1(R^n)$$

$$B = \begin{pmatrix} \gamma & 0 & 0 \\ 0 & \gamma & 0 \\ 0 & 0 & \gamma \end{pmatrix}, \ D(B) = \mathcal{H},$$

$$F(w) = \begin{bmatrix} |u_1|^{p_1-1} |u_2|^{p_2+1} |u_3|^{p_3+1} u_1 \\ |u_1|^{p_1+1} |u_2|^{p_2-1} |u_3|^{p_3+1} u_2 \\ |u_1|^{p_1+1} |u_2|^{p_2+1} |u_3|^{p_3-1} u_3 \end{bmatrix}.$$

A is a self-adjoint positive definite operator, and conditions (2.3), (2.4) imply that $F(w)$ is a nonlinear operator acting from $\mathcal{H}_1 = H^1 \times H^1 \times H^1$ to $\mathcal{H}$.

Lemma 2.1 *Let conditions* (2.1) *and* (2.2) *be satisfied. Then the nonlinear operator $\mathcal{H}_1 \to \mathcal{H}$ satisfies the local Lipschitz condition, that is for any $w^1, w^2 \in \mathcal{H}_1$ the following inequality is satisfied*

$$\left\| F\left(w^1\right) - F\left(w^2\right) \right\|_{\mathcal{H}} \le c(r) \left\| w^1 - w^2 \right\|_{\mathcal{H}_1},$$

where $c(\cdot) \in C(R_+)$, $c(r) \ge 0$, $r = \sum\limits_{i=1}^{3} \left\| w^i \right\|_{\mathcal{H}_1}$.

By the theorem of solvability of the Cauchy problem for the operator differential equation, we have the following local solvability theorem for problem (2.3),(2.4) (see [3]).

Theorem 2.1 *Let conditions* (2.1) *and* (2.2) *be satisfied. Then there exists a $T' > 0$ such that for arbitrary $w_0 \in \mathcal{H}_1$, $w_1 \in \mathcal{H}$, problem* (2.3), (2.4) *has a unique solution*

$$w(\cdot) \in C([0, T^*); \mathcal{H}_1) \bigcap C^1([0, T^*); \mathcal{H}).$$

If $T_{\max} = \sup T^*$, i.e., $T_{\max}$ is the length of the maximal existence interval of the solution $w(\cdot) \in C([0, T_{\max}); \mathcal{H}_1) \bigcap C^1([0, T_{\max}); \mathcal{H})$, then either

(i) $T_{\max} = +\infty$,

or

(ii) $\limsup\limits_{t \to T_{\max} - 0} \left[\|w(\cdot)\|_{\mathcal{H}_1} + \|\dot{w}(\cdot)\|_{\mathcal{H}} \right] = +\infty$.

Remark 2.1 If $w_0 \in \mathcal{H}_2$, $w_1 \in \mathcal{H}_1$, then $w(\cdot) \in C\left([0, T_{\max}); \mathcal{H}_2\right) \bigcap C^1\left([0, T_{\max}); \mathcal{H}_1\right)$, where
$\mathcal{H}_2 = H^2 \times H^2 \times H^2$.

Let us denote by $E(t)$ the energy function

$$E(t) = \sum_{j=1}^{3} \frac{p_j + 1}{2} \left[\left| u'_{jt}(t, \cdot) \right|^2 + \| u_j(t, \cdot) \|^2 + 2\gamma \int_0^t \left| u'_{jt}(s, \cdot) \right|^2 ds \right]$$

$$- \int_{R^n} |u_1(t, x)|^{p_1+1} \cdot |u_2(t, x)|^{p_2+1} |u_3(t, x)|^{p_3+1} dx.$$

Introduce also the following notation

$$I(\phi_1, \phi_2, \phi_3) = \sum_{j=1}^{3} \frac{p_j + 1}{p_1 + p_2 + p_3 + 3} \|\phi_j\|^2$$

$$- \int_{R^n} |\phi_1(t, x)|^{p_1+1} \cdot |\phi_2(t, x)|^{p_2+1} |\phi_3(x)|^{p_3+1} dx, \ \ \phi_i \in H^1, \ \ i = 1, 2, 3. \tag{2.5}$$

The main result of the present paper is stated in the following statement.

Theorem 2.2 *Let conditions* (2.1) *and* (2.2) *be satisfied, let* $u_{i0} \in H^1$ *and* $u_{i1} \in L_2(R^n)$, $i = 1, 2, 3$ *and in addition, let the following conditions be satisfied:*

$$E(0) > 0, \tag{2.6}$$

$$I(u_{10}, u_{20}, u_{30}) < 0, \tag{2.7}$$

$$\sum_{j=1}^{3} \langle u_{j0}, u_{j1} \rangle \geq 0, \tag{2.8}$$

$$\sum_{j=1}^{3} \frac{p_j + 1}{2} |u_{j0}|^2 > \frac{p_2 + p_2 + p_3 + 3}{p_2 + p_2 + p_3} E(0). \tag{2.9}$$

Then the solution of the Cauchy problem (1.1), (1.2) *blows up in finite time.*

3 Some auxiliary assertions

Lemma 3.1 *Let conditions* (2.6) - (2.8) *be satisfied. Then following inequality holds*

$$I(u_1(t, \cdot), u_2(t, \cdot), u_3(t, \cdot)) < 0, \quad t \in [0, T_{\max}).$$

Proof. By virtue of (2.7) there exists $T_1 > 0$, such that

$$I(u_1(t, \cdot), u_2(t, \cdot), u_3(t, \cdot)) < 0, t \in [0, T_1). \tag{3.1}$$

We shall prove that $T_1 = T_{\max}$. Let $T_1 < T_{\max}$, then

$$I(u_1(T_1, \cdot), u_2(T_1, \cdot), u_3(T_1, \cdot)) = 0. \tag{3.2}$$

We introduce the functional $F(t) = \sum_{j=1}^{3} (p_j + 1) |u_j(t,\cdot)|^2$. Taking into account Remark 2.1 and using (1.1), (1.2) we obtain:

$$F'(t) = 2\sum_{j=1}^{3} (p_j + 1) \langle u_j(t,\cdot), \dot{u}_j(t,\cdot)\rangle ,$$

and

$$F''(t) = 2\sum_{j=1}^{3} (p_j + 1) \left|u_j'(t,\cdot)\right|^2 - 2\sum_{j=1}^{3} (p_j + 1) \left[\|u_j(t,\cdot)\|^2 + \gamma \langle u_j(t,\cdot), \dot{u}_j(t,\cdot)\rangle\right]$$

$$+2(p_1 + p_2 + p_3 + 3) \int_{R^n} |u_1(t,x)|^{p_1+1} \cdot |u_2(t,x)|^{p_2+1} |u_3(t,x)|^{p_3+1} dx.$$

Therefore,

$$F''(t) + \gamma F'(t) = \varphi(t), t \in [0, T_1) \tag{3.3}$$

where

$$\varphi(t) = 2\sum_{j=1}^{3} (p_j + 1) \left|u_j'(t,\cdot)\right|^2 - 2(p_1 + p_2 + p_3 + 3) I(u_1(t,\cdot), u_2(t,\cdot), u_3(t,\cdot)).$$

By inequality (3.1) we have

$$\varphi(t) > 0, t \in [0, T_1). \tag{3.4}$$

It follows from condition (2.7) and relations (3.3) and (3.4) that

$$F'(t) > 0, t \in [0, T_1).$$

Therefore, the function F(t) is monotone increasing on $t \in [0, T_1)$: consequently

$$F(t) > F(0) = \sum_{j=1}^{3} (p_j + 1) |u_{j0}|^2. \tag{3.5}$$

By taking into account the continuity of the function $F(t)$, from condition (2.9) and inequalities (3.5) we obtain

$$F(T_1) > \frac{(2p_1 + p_2 + p_3 + 3)}{p_1 + p_2 + p_3} E(0). \tag{3.6}$$

On the other hand, relation (3.1) and the definition of the function $E(t)$ imply the inequality

$$\left(1 - \frac{2}{p_1 + p_2 + p_3 + 3}\right) \sum_{j=1}^{3} \frac{p_j + 1}{2} \|u_j(t,\cdot)\|^2 \le E(0), \quad t \in [0, T_1).$$

It follows that

$$F(t) \le \frac{(2p_1 + p_2 + p_3 + 3)}{p_1 + p_2 + p_3} E(0), \quad t \in [0, T_1). \tag{3.7}$$

The resulting contradiction (3.6), (3.7) shows that our assumption fails. Therefore $T_1 = T_{\max}$.

Let $T_2 > 0$ $T_3 > 0$, and $k > 0$ be some positive numbers. Consider the functional

$$R(t)$$

$$= \sum_{j=1}^{3} \frac{p_j+1}{2} \left[|u_j(t,\cdot)|^2 + \gamma \int_0^t |u_j(s,\cdot)|^2 ds + \gamma |u_{j0}|^2 (T_2 - t) \right] + k(T_3+t)^2 . \quad (3.8)$$

Lemma 3.2 *Let conditions* (2.6) - (2.9) *be satisfied. Then the following inequality*

$$R''(t) > 0, \quad t \in [0, T_{\max})$$

holds.

Proof. A simple computation will give us

$$R'(t)$$

$$= \sum_{j=1}^{3} \frac{p_j+1}{2} \left[2 \langle u_j(t,\cdot), u_j'(t,\cdot) \rangle + \gamma |u_j(t,\cdot)|^2 - \gamma |u_{j0}|^2 \right] + 2k(t+T_3) . \quad (3.9)$$

Next, from (3.9), by using relations (1.1) and (1.2), we obtain

$$R''(t) = \sum_{j=1}^{3} (p_j+1) \left[|u_j'(t,\cdot)|^2 - \|u_j(t,\cdot)\|^2 \right]$$

$$+ (p_1+p_2+p_3+3) \int_{R^n} |u_1(t,x)|^{p_1+1} \cdot |u_2(t,x)|^{p_2+1} |u_3(t,x)|^{p_3+1} dE + 2k. \quad (3.10)$$

It follows from (2.5) and (3.10) that

$$R''(t) \geq -(p_1+p_2+p_3+3) \cdot I(u_1(t,.), u_2(t,.).u_3(t,.)) + 2k, \quad t \in [0, T_{\max}) .$$

By virtue of Lemma 3.2 for sufficiently small k it takes place

$$R''(t) > 0, t \in [0, T_{\max}) . \quad (3.11)$$

4 Proof of main theorem

We first assume that $u_{i0} \in H^2$, $u_{i1} \in H^1$, $i = 1,2,3$. We shall prove that under the conditions (2.6) - (2.9) $T_{\max} < +\infty$. Suppose the contrary: $T_{\max} = +\infty$.

It follows from (1.1) and (1.2) that

$$\int_{R^n} |u_1(t,x)|^{p_1+1} \cdot |u_2(t,x)|^{p_2+1} |u_3(t,x)|^{p_3+1} dx = -E(0)$$

$$+ \sum_{j=1}^{3} \frac{p_j+1}{2} \left[|u_j'(t,\cdot)|^2 + \|u_j(t,\cdot)\|^2 + 2\gamma \int_0^t |u_j'(s,\cdot)|^2 ds \right]$$

and by taking into account this relation in (3.10), we obtain

$$R''(t) = \frac{p_1+p_2+p_3+5}{2}\sum_{j=1}^{3}(p_j+1)\left|u_j'(t,\cdot)\right|^2$$

$$+\frac{p_1+p_2+p_3+1}{2}\sum_{j=1}^{3}(p_j+1)\left\|u_j(t,\cdot)\right\|^2+\gamma(p_1+p_2+p_3+3)$$

$$\times\sum_{j=1}^{3}(p_j+1)\int_0^t\left|u_j'(s,\cdot)\right|^2ds-(p_1+p_2+p_3+3)E(0)+2k. \tag{4.1}$$

From (3.9) we have

$$R'^2(t)\le\left[\sum_{j=1}^{3}(p_j+1)\left(\left|u_j(t,\cdot)\right|^2+\gamma\int_0^t\left|u_j(s,\cdot)\right|^2ds\right)+k(t+T_3)^2\right]$$

$$\times\left[\sum_{j=1}^{3}(p_j+1)\left(\left|u_j'(t,\cdot)\right|^2+\gamma\int_0^t\left|u_j'(s,\cdot)\right|^2ds\right)+k\right]. \tag{4.2}$$

By choosing a sufficiently large T_3, from Lemma 3.2 and relations (3.10), (4.1), and (4.2) we obtain

$$R(t)\cdot R''(t)-\frac{p_1+p_2+p_3+5}{4}R'^2(t)$$

$$\ge R(t)\cdot R''(t)-\frac{p_1+p_2+p_3+5}{4}\left[2R(t)-(T_1-t)\sum_{j=1}^{3}(p_j+1)\cdot\left|u_{j0}\right|^2\right]$$

$$\times\left[\sum_{j=1}^{3}(p_j+1)\left(\left|u_j'(t,\cdot)\right|^2+\gamma\int_0^t\left|u_j'(s,\cdot)\right|^2ds\right)+k\right]$$

$$\ge R(t)\left\{\frac{p_1+p_2+p_3+3}{2}\sum_{j=1}^{3}(p_j+1)\left|u_j'(t,\cdot)\right|^2\right.$$

$$+\frac{p_1+p_2+p_3+1}{2}\sum_{j=1}^{3}(p_j+1)\left\|u_j(t,\cdot)\right\|^2$$

$$+(p_1+p_2+p_3+3)\sum_{j=1}^{3}(p_j+1)\int_0^t\left|u_j'(s,\cdot)\right|^2ds-(p_1+p_2+p_3+3)E(0)+2k$$

$$\left.-\frac{p_1+p_2+p_3+5}{2}\sum_{j=1}^{3}(p_j+1)\left(\left|u_j'(t,\cdot)\right|^2+\int_0^t\left|u_j'(s,\cdot)\right|^2ds\right)+k\right\}$$

$$=R(t)\,y(t)+\frac{p_1+p_2+p_3+1}{2}\sum_{j=1}^{3}\int_0^t\left|u_j'(s,.)\right|^2ds, \tag{4.3}$$

where

$$y(t) = \frac{p_1 + p_2 + p_3 + 1}{2} \sum_{j=1}^{3} (p_j + 1) \|u_j(t, \cdot)\|^2$$

$$- (p_1 + p_2 + p_3 + 3) E(0) - \frac{p_1 + p_2 + p_3 + 1}{2} k.$$

Having in mind Lemma 3.1, choosing a sufficiently small $k > 0$, we obtain that $y(t) \geq 0$.Thus, for sufficiently large $T_2 > 0$,$T_3 > 0$, and for sufficiently small $k > 0$ the following inequality

$$R(t) \cdot R''(t) - \frac{p_1 + p_2 + p_3 + 5}{4} R'^2(t) \geq 0 \tag{4.4}$$

holds.

On the other hand,

$$R'(0) = \sum_{j=1}^{m} (p_j + 1) \langle u_{j0}, u_{j1} \rangle + 2kT_2 .$$

Therefore

$$R'(0) > 0. \tag{4.5}$$

Next, using (4.4) and (4.5) by the standard procedure we obtain that there exists $0 < T^* < +\infty$ such that $\lim\limits_{t \to T^* - 0} R(t) = +\infty$.

Resulting contradiction shows that $T_{\max} < +\infty$.

If $u_{i0} \in H^1$ and $u_{i1} \in L_2(R^n)$, $i = 1, 2, 3$, then the justification can be carried out in a standard way, by approximation of the initial data by functions from H^2 and H^1 respectively.

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