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Soft countable topological spaces

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Abstract. *In 1999, Russian researcher Molodtsov proposed the new concept of a soft set which can be considered as a new mathematical approach for vagueness. Soft topological spaces have been studied by some authors in recent years. The purpose of this paper is to investigate some concepts of soft topological spaces which is a generalization to the notion of topological spaces. We give some new concepts in soft topological spaces such as soft first-countable spaces, soft second-countable spaces and soft separable spaces, soft sequential continuity and some important properties of these concepts are investigated.*

Keywords. Soft set, soft point,soft A_1-space, soft A_2-space, soft separable space and soft sequentially continuity.

Mathematics Subject Classification (2010): 54A05, 54A20

1 Introduction

The soft set theory, initiated by Russian researcher D. Molodtsov [15], is one of the branches of mathematics, which aims to describe phenomena and concepts of an ambiguous, vague, undefined and imprecise meaning. Also soft set theory is applicable where there is no clearly defined mathematical model. Since soft set theory has a rich potential, researchs on soft set theory and its applications in various fields are progressing rapidly in [2],[13]. The algebraic structure of set theories is an important problem. Aktas and Cagman [3] defined soft groups and derived their basic properties. U. Acar et al.[1] introduced initial concepts of soft rings. F. Feng et al. [7] defined soft semirings and several related notions to establish a connection between soft sets and semirings. C. Gunduz and S. Bayramov [8],[9] introduced fuzzy soft modules and intuitionistic fuzzy soft modules and investigated some important properties of these modules. T. Y. Ozturk and S. Bayramov [17] defined chain complexes of soft modules and soft homology modules of them. T. Y. Ozturk et al. [18] introduced the concept of inverse and direct systems in the category of soft modules.

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Topological stuructures of soft set have been studied by some authors in recent years. M. Shabir and M. Naz [19] have initiated the study of soft topological spaces which are defined over an initial universe with a fixed set of parameters and showed that a soft topological space gives a parameterized family of topological spaces. Theoretical studies of soft topological spaces have also been researched by some authors in [4], [5], [10], [11], [14], [16], [20], [22]. By using soft points, S.Das and S.K.Samanta [6] define the concept of soft metric. Later M.I.Yazar et al. [21] examined some important properties of soft metric spaces and proved some fixed point theorems of soft contractive mappings on soft metric spaces.

The purpose of this paper is to investigate some concepts of soft topological spaces. We give some new concepts in soft topological spaces such as soft first-countable spaces, soft second-countable spaces and soft separable spaces, soft sequential continuity and also some important properties of these concepts are investigated.

2 Preliminaries

In this section we will introduce necessary definitions and theorems for soft sets. Throughout this paper X denotes initial universe, E denotes the set of all parameters, $P(X)$ denotes the power set of X.

Definition 2.1 *[15] A pair (F, E) is called a soft set over X, where F is a mapping given by $F : E \to P(X)$.*

In other words, the soft set is a parameterized family of subsets of the set X. For $e \in E$, $F(e)$ may be considered as the set of $e-$elements of the soft set (F, E), or as the set of $e-$approximate elements of the soft set.

After this, $SS(X)_E$ denotes the family of all soft sets over X with a fixed set of parameters E.

Definition 2.2 *[2] For two soft sets (F, E) and (G, E) over X, (F, E) is called a soft subset of (G, E) if $\forall e \in E$, $F(e) \subseteq G(e)$. This relationship is denoted by $(F, E)\widetilde{\subseteq}(G, E)$.*

Similarly, (F, E) is called a soft superset of (G, E) if (G, E) is a soft subset of (F, E). This relationship is denoted by $(F, E)\widetilde{\supseteq}(G, E)$. Two soft sets (F, E) and (G, E) over X are called soft equal if (F, E) is a soft subset of (G, E) and (G, E) is a soft subset of (F, E).

Definition 2.3 *[2] The intersection of two soft sets (F, E) and (G, E) over X is the soft set (H, E), where $\forall e \in E$, $H(e) = F(e) \cap G(e)$. This is denoted by $(F, E)\widetilde{\cap}(G, E) = (H, E)$.*

Definition 2.4 *[2] The union of two soft sets (F, E) and (G, E) over X is the soft set (H, E), where $\forall e \in E$, $H(e) = F(e) \cup G(e)$. This is denoted by $(F, E)\widetilde{\cup}(G, E) = (H, E)$.*

Definition 2.5 *[13] A soft set (F, E) over X is said to be a null soft set denoted by Φ if for all $e \in E$, $F(e) = \varnothing$.*

Definition 2.6 *[13] A soft set (F, E) over X is said to be an absolute soft set denoted by $\widetilde{X}$ if for all $e \in E$, $F(e) = X$.*

Definition 2.7 *[19] The difference (H, E) of two soft sets (F, E) and (G, E) over X, denoted by $(F, E)\backslash(G, E)$, is defined as $H(e) = F(e)\backslash G(e)$ for all $e \in E$.*

Definition 2.8 *[19] The complement of a soft set (F, E), denoted by $(F, E)^c$, is defined $(F, E)^c = (F^c, E)$, where $F^c : E \to P(X)$ is a mapping given by $F^c(e) = X\backslash F(e)$, $\forall e \in E$ and F^c is called the soft complement function of F.*

Definition 2.9 *[12] Let (X,E) and $(Y,E^{'})$ be two soft sets, $f:X\to Y$ and $g:E\to E^{'}$ be two mappings. Then $(f_g):(X,E)\to(Y,E^{'})$ is called a soft mapping and is defined as: for a soft set (F,A) in (X,E), $(f_g)((F,A))=f(F)_{g(a)}$, $B=g(A)\subseteq E^{'}$ is a soft set in $(Y,E^{'})$ given by*

$$f(F)(e')=\begin{cases} f\left(\underset{e\in g^{-1}(e')\cap A}{\cup}F(e)\right), & \text{if } g^{-1}(e')\cap A\neq\emptyset,\\ \emptyset, & \text{otherwise,}\end{cases}$$

for $e'\in B\subseteq E^{'}$. $(f(F),g(A))$ is called a soft image of a soft set (F,A).

Definition 2.10 *[12] Let (X,E) and $(Y,E^{'})$ be two soft sets, $(f_g):(X,E)\to(Y,E^{'})$ be a soft mapping and $(G,C)\widetilde{\subseteq}(Y,E^{'})$. Then $(f_g)^{-1}((G,C))=f^{-1}(G)_{g^{-1}(C)}$, $D=g^{-1}(C)$, is a soft set in the soft set (X,E), defined as:*

$$f^{-1}(G)(e)=\begin{cases} f^{-1}\left(G(g(e))\right), & \text{if } g(e)\in C,\\ \emptyset, & \text{otherwise,}\end{cases}$$

for $e\in D\subseteq E$. $(f_g)^{-1}((G,C))$ is called a soft inverse image of (G,C).

Definition 2.11 *[19] Let τ be the collection of soft sets over X, then τ is said to be a soft topology on X if*

1) $\Phi,\widetilde{X}$ belongs to τ,
2) the union of any number of soft sets in τ belongs to τ,
3) the intersection of any two soft sets in τ belongs to τ.

The triplet (X,τ,E) is called a soft topological space over X.

Definition 2.12 *[19] Let (X,τ,E) be a soft topological space over X, then members of τ are said to be soft open sets in X.*

Definition 2.13 *[19] Let (X,τ,E) be a soft topological space over X. A soft set (F,E) over X is said to be a soft closed set in X, if its complement $(F,E)^c$ belongs to τ.*

Proposition 2.1 *[19] Let (X,τ,E) be a soft topological space over X. Then the collection $\tau_e=\{F(e):(F,E)\in\widetilde{\tau}\}$ for each $e\in E$, defines a topology on X.*

Definition 2.14 *[19] Let (X,τ,E) be a soft topological space over X and (F,E) be a soft set over X. Then the soft closure of (F,E), denoted by $\overline{(F,E)}$, is the intersection of all soft closed super sets of (F,E). Clearly $\overline{(F,E)}$ is the smallest soft closed set over X which contains (F,E).*

Definition 2.15 *[19] Let (X,τ,E) be a soft topological space over X and (F,E) be a soft set over X. Then the soft interior of (F,E), denoted by $(F,E)^0$, is the union of all soft open subsets of (F,E).*

Definition 2.16 *[4] Let (F,E) be a soft set over X. The soft set (F,E) is called a soft point, denoted by (x_e,E), if for the element $e\in E$, $F(e)=\{x\}$ and $F(e')=\varnothing$ for all $e'\in E-\{e\}$ (briefly denoted by x_e).*

It is obvious that each soft set can be expressed as a union of soft points. For this reason, to give the family of all soft sets on X it is sufficient to give only soft points on X.

Definition 2.17 *[4] Two soft points x_e and $y_{e'}$ over a common universe X, we say that the soft points are different if $x\neq y$ or $e\neq e'$.*

Definition 2.18 *[4] The soft point x_e is said to be belonging to the soft set (F,E), denoted by $x_e\widetilde{\in}(F,E)$, if $x_e(e)\in F(e)$,i.e., $\{x\}\subseteq F(e)$.*

Definition 2.19 *[4] Let (X,τ,E) be a soft topological space over X. A soft set (F,E) $\widetilde{\subseteq}(X,E)$ is called a soft neighborhood of the soft point $x_e\widetilde{\in}(F,E)$ if there exists a soft open set (G,E) such that $x_e\widetilde{\in}(G,E)\widetilde{\subset}(F,E)$.*

Definition 2.20 *[10] Let (X,τ,E) and (Y,τ',E) be two soft topological spaces and $f:(X,\tau,E)\to(Y,\tau',E)$ be a soft mapping. For each soft neighborhood (H,E) of $(f(x))_e$, if there exists a soft neighborhood (F,E) of x_e such that $f((F,E))\widetilde{\subset}(H,E)$, then f is said to be soft continuous mapping at x_e.*

If f is a soft continuous mapping for all x_e, then f is called soft continuous mapping.

Definition 2.21 *[10] Let (X,τ,E) and (Y,τ',E) be two soft topological spaces and $f:(X,\tau,E)\to(Y,\tau',E)$ be a soft mapping. If f is a soft bijection, soft continuous and $f^{-1}$is a soft continuous mapping, then f is said to be soft homeomorphism from X to Y. When a soft homeomorphism f exists between (X,τ,E) and (Y,τ',E), we say that (X,τ,E) is soft homeomorphic to (Y,τ',E).*

3 Soft countable topological spaces

Let (X,τ,E) be a soft topological space, x_e be a soft point, $\grave{U}(x_e)$ be a family of soft neighborhood and $B(x_e)\subset\grave{U}(x_e)$.

Definition 3.1 *Let (X,τ,E) be a soft topological space. A subcollection $B\subset\tau$ is called a soft base for τ if every member of τ can be written as a union of members of B.*

Definition 3.2 *Let (X,τ,E) be a soft topological space and $\grave{U}(x_e)$ be a family of soft neighborhood of soft point x_e. If there exists $(G,E)\in B(x_e)$ such that $x_e\in(G,E)\subset(F,E)$, then the family $B(x_e)$ is called a soft neighborhood base of x_e, for each soft neighborhood $(F,E)\in\grave{U}(x_e)$.*

Proposition 3.1 *Let (X,τ,E) be a soft topological space, $B\subset\tau$. The family B is a soft base of τ if and only if the family $\widetilde{B}(x_e)=\{(G,E)\in B:\ x_e\in(G,E)\}$ is a soft neighborhood base of x_e.*

Proof. Assume that let B be a soft base of τ and x_e be a soft point. The members of $\widetilde{B}(x_e)$ is a soft open neighborhood of x_e. If $(F,E)\in\grave{U}(x_e)$, then $(F,E)=\bigcup_{(G,E)\in B}(G,E)$ is satisfied. Thus $x_e\in(G,E)\subset(F,E)$ and $(G,E)\in\widetilde{B}(x_e)$ is obtained, i.e, $\widetilde{B}(x_e)$ is a soft neighborhood base of x_e.

Conversely, let $\widetilde{B}(x_e)=\{(G,E)\in B:\ x_e\in(G,E)\}$ be a soft neighborhood base for each soft point x_e. If $(F,E)\in\tau$, then there exists $(G,E)_{x_e}\in\widetilde{B}(x_e)$ such that $x_e\in(G,E)_{x_e}\subset(F,E)$ for each $x_e\in(F,E)\in\grave{U}(x_e)$. Hence $(F,E)=\bigcup_{x_e\in(F,E)}\{x_e\}\subset\bigcup_{x_e\in(F,E)}(G,E)_{x_e}\subset(F,E)$, i.e., $(F,E)=\bigcup_{x_e\in(F,E)}(G,E)_{x_e}$ is obtained. Thus the family $B=\bigcup_{x_e}\widetilde{B}(x_e)$ is a soft base of τ.

Proposition 3.2 *Let (X,τ,E) be a soft topological space and x_e be a soft point. Then $\widetilde{B}(x_e)$ has the following properties:*

a) If $(G,E) \in \widetilde{B}(x_e)$ *, then* $x_e \in (G,E)$*,*

b) If $(G_1,E),(G_2,E) \in \widetilde{B}(x_e)$ *, then there exists* $(F,E) \in \widetilde{B}(x_e)$ *such that* $(F,E) \subset (G_1,E) \cap (G_2,E)$*.*

c) If $(G,E) \in \widetilde{B}(x_e)$ *, there exists* $(V,E) \in \widetilde{B}(x_e)$ *such that, for each* $y_{e'} \in (V,E)$*, there is* $(W,E) \in \widetilde{B}(y_{e'})$ *,* $y_{e'} \in (W,E) \subset (G,E)$*.*

Proof. The proof of proposition is clear.

Definition 3.3 *Let* (X,τ,E) *be a soft topological space and* x_e *be a soft point.*

a) If there exists a countable soft neighborhood base for each soft point x_e, then (X,τ,E) is called a soft first countable space and denoted by soft A_1-space.

b) If (X,τ,E) has a countable soft base, then (X,τ,E) is called a soft second countable space and denoted by soft A_2-space.

c) If some countable soft subset of (X,τ,E) is dense in (X,τ,E), then (X,τ,E) is called a soft separable space.

Theorem 3.1 *Let* (X,τ,E) *be a soft topological space and* x_e *be a soft point. If* (X,τ,E) *is a soft* A_1*-space, then there exists a countable soft open neighborhoods base* $V(x_e) = \{(V_n,E)\}_{n\in N}$ *of soft point* x_e *such that* $(V_{n+1},E) \subset (V_n,E)$ *for each* $n \in N$*.*

Proof. Let (X,τ,E) be a soft A_1-space and $\widetilde{B}(x_e) = \{(B_n,E)\}_{n\in N}$ be a soft neighborhood base of soft point x_e. If we take as $(V_n,E) = (B_1,E)^\circ \cap (B_2,E)^\circ \cap ... \cap (B_n,E)^\circ$ for each $n \in N$, (V_n,E) are soft open sets and $(V_{n+1},E) \subset (V_n,E)$ is satisfied. Since $\widetilde{B}(x_e)$ is a soft neighborhood base, there exists $m \in N$ such that $x_e \in (B_m,E) \subset (U,E)$ for each soft neighborhood (U,E) of soft point x_e. Thus $x_e \in (V_m,E) \subset (B_m,E) \subset (U,E)$ is obtained i.e., $V(x_e) = \{(V_n,E)\}_{n\in N}$ is a countable soft open neighborhoods base.

Theorem 3.2 *Let* (X,τ,E) *be a soft topological space. If* (X,τ,E) *is a soft* A_2*-space, then* (X,τ,E) *is also a soft* A_1*-space.*

Proof. *It is obvious.*

Theorem 3.3 *Let* (X,τ,E) *be a soft topological space. If* (X,τ,E) *is a soft* A_2*-space, then* (X,τ,E) *is a soft separable space.*

Proof. Let (X,τ,E) be a soft A_2-space, and let $B = \{(B_n,E)\}_{n\in N}$ be a countable base of (X,τ,E). If we choose $x^n_{e_n} \in (B_n,E)$ for each $n \in N$, then the soft set $(M,E) = \{x^n_{e_n}\}_{n\in N}$ is obtained. For each $(G,E) \in \tau$ and $x_e \in (G,E)$, there exists soft set (B_n,E) such that $x_e \in (B_n,E) \subset (G,E)$. Hence $x^n_{e_n} \in (M,E) \cap (B_n,E) \subset (M,E) \cap (G,E) \neq \Phi$ is obtained i.e., (X,τ,E) is a soft separable space.

Remark 3.1 If (X,τ,E) is a soft A_1-space, then (X,τ_e) is a A_1-space, for each $e \in E$.

Remark 3.2 If (X,τ,E) is a soft A_2-space, then (X,τ_e) is a A_2-space, for each $e \in E$.

Theorem 3.4 *Let* (X,τ,E) *be a soft topological space. If* (X,τ,E) *is a soft separable metric space, then* (X,τ,E) *is a soft* A_2*-space.*

Proof. Let $\left(X,\widetilde{d},E\right)$ be a soft metric space, and let (F,E) be a countable dense soft set. For each $x_e \in (F,E)$ and $r \in Q, r > 0$, take soft open ball $B(x_e,r)$. Now, we show that the family ß= $\{B(x_e,r) : x_e \in (F,E), r \in Q, r > 0\}$ is a soft base. For each $(G,E) \in \tau_{\widetilde{d}}$ and $y^o_{e_o} \in (G,E)$, the soft ball $y^o_{e_o} \in B\left(y^o_{e_o},\varepsilon\right) \subset (G,E)$ is obtained. Since (F,E) is a dense soft set in $\left(X,\widetilde{d},E\right)$, there exists $x_e \in (F,E)$ such that $x_e \in B\left(y^o_{e_o},\frac{\varepsilon}{3}\right)$. Hence, for $\frac{\varepsilon}{3} < r < \frac{2\varepsilon}{3}$ and $r \in Q$, $\widetilde{d}\left(y^o_{e_o},x_e\right) < \frac{\varepsilon}{3} < r$ is satisfied. For each $z_{e_1} \in B(x_e,r)$, $\widetilde{d}\left(y^o_{e_o},z_{e_1}\right) < \varepsilon$ is obtained. Thus $B(x_e,r)$ is the soft ball such that $z_{e_1} \in B(x_e,r) \subset (G,E)$.

Definition 3.4 *Let (X,τ,E) be a soft topological space, $\left\{x^n_{e_n}\right\}$ be a soft sequence and $x^0_{e_0}$ be a soft point. The sequence $\left\{x^n_{e_n}\right\}$ is said to converge to the soft point $x^0_{e_0}$ if there exists $n_0 \in \mathbb{N}$, for all $n \geq n_0$ such that $x^n_{e_n} \in (U,E)$ for each soft neighborhood (U,E) of soft point x_e, denoted by $\lim\limits_{n\to\infty} x^n_{e_n} = x^0_{e_0}$.*

Definition 3.5 *Let (X,τ,E) be a soft topological space, $\left\{x^n_{e_n}\right\}$ be a soft sequence and $x^0_{e_0}$ be a soft point. The soft point $x^0_{e_0}$ is said to an accumulation point of $\left\{x^n_{e_n}\right\}$ if there exists $m \geq n$ such that $x^m_{e_m} \in (U,E)$ for each soft neighborhood (U,E) of soft point x_e and each $n \in \mathbb{N}$.*

Now let us examine the following examples in soft topological space. The result just established has an application to converge of soft sequences.

Example 1 Let $\mathbb{R}$ be real line and $X = \mathbb{R}$ and $E = \{a,b\}$, $a,b \in \mathbb{R}^+$ and $|a-b| = 1$. Then $\widetilde{d}(x_a,y_b) = |a-b| + |x-y|$ is a soft metric on X and $\left(X,\widetilde{d},E\right)$ is a soft metric space. Let us consider sequence of soft points as

$$\left\{\left(\frac{1}{2}\right)_a,\left(\frac{1}{4}\right)_b,\left(\frac{1}{8}\right)_a,\left(\frac{1}{16}\right)_b,\ldots\right\}.$$

The soft sequence does not converge to any soft point in $\left(X,\widetilde{d},E\right)$. If we fixed the parameters a and b, then the sequence $\left\{\frac{1}{2},\frac{1}{8},\ldots\right\}$ converges to in the metric space (X,d_a) and the sequence $\left\{\frac{1}{4},\frac{1}{16},\ldots\right\}$ converges to in the metric space (X,d_b). Thus given every sequence converges according to every parameter, but the sequence of soft points doesn't converge.

Example 2 Let $X = \mathbb{R}$ and $E = [0,1]$ be closed interval. Then $\widetilde{d}(x_e,y_{e'}) = |e-e'| + |x-y|$ is a soft metric on X and $\left(X,\widetilde{d},E\right)$ is a soft metric space. Let us consider sequence as $\left\{\left(\frac{1}{2^n}\right)_{\frac{1}{n}}\right\}_{n\in\mathbb{N}}$. The soft sequence converges to soft point 0_0. But there are no soft points in this sequence with parameter 0.

Example 3 Let us consider the following soft sequence

$$\left\{1_a,\left(\frac{1}{3}\right)_a,\ldots,\left(\frac{1}{3^{100}}\right)_a,\left(\frac{1}{3^{101}}\right)_b,\left(\frac{1}{3^{102}}\right)_b,\ldots\right\}.$$

The soft sequence converges to soft point 0_b. If we fixed the parameter a, then the sequence does not converge.

Theorem 3.5 *Let (X,τ,E) be a soft A_1-space, $\left\{x^n_{e_n}\right\}$ be a soft sequence and $x^0_{e_0}$ be a soft point. The soft point $x^0_{e_0}$ is an accumulation point of $\left\{x^n_{e_n}\right\}$ if and only if the soft sequence $\left\{x^n_{e_n}\right\}$ has a soft subsequence which converges to $x^0_{e_0}$.*

Proof. Sufficiency: Let (X, τ, E) be a soft A_1-space and $\left\{x_{e_n}^n\right\}$ be a soft sequence. Since (X, τ, E) is a soft A_1-space, then there exists a countable soft neighborhood base as

$$\{(V_n, E) : (V_{n+1}, E) \subset (V_n, E)\} \tag{3.1}$$

of soft point $x_{e_0}^0$. Also since the soft point $x_{e_0}^0$ is an accumulation point, we choose soft subsequence the following form:

for (V_1, E) and 1$\in \mathbb{N}$, there exists $n_1 > 1$ such that $x_{e_{n_1}}^{n_1} \in (V_1, E)$,

for (V_2, E) and $n_1 \in \mathbb{N}$, there exists $n_2 > n_1$ such that $x_{e_{n_2}}^{n_2} \in (V_2, E)$,

....

for (V_k, E) and $n_{k-1} \in \mathbb{N}$, there exists $n_k > n_{k-1}$ such that $x_{e_{n_k}}^{n_k} \in (V_k, E)$,

....

Thus there exists $k_0 \in \mathbb{N}$ such that $(V_{k_0}, E) \subset (U, E)$ of each soft neighborhood (U, E) . Hence for each $k \geq k_0$, $x_{e_{n_k}}^{n_k} \in (V_k, E) \subset (V_{k_0}, E) \subset (U, E)$ is satisfied, i.e., $\lim\limits_{k\to\infty} x_{e_{n_k}}^{n_k} = x_{e_0}^0$ is obtained.

Necessity: Let $\left\{x_{e_{n_k}}^{n_k}\right\}$ be a soft subsequence which converges to $x_{e_0}^0$ of the soft sequence $\left\{x_{e_n}^n\right\}$. For each soft neighborhood (U, E) of soft point $x_{e_0}^0$, there exists $k_0 \in \mathbb{N}$ such that for all $k \geq k_0, x_{e_{n_k}}^{n_k} \in (U, E)$ is obtained. Then it is easy to see that there exists $n_m \in \mathbb{N}$ such that $n_m > n$, for each $n \in \mathbb{N}$. If we take $j = \max\{m, k_0\}$, then $j \geq k_0$. Hence $x_{e_{n_j}}^{n_j} \in (U, E)$ and $n_j \geq n_m > n$ are satisfied. i.e., the soft point $x_{e_0}^0$ is an accumulation point of $\left\{x_{e_n}^n\right\}$.

Remark 3.3 Let (X, τ, E) be a soft topological space. If the sequence $\{x^n\} \subset (X, \tau_e)$ converges to point x^0, then the soft sequence $\{x_e^n\}$ converges to soft point x_e^0 in (X, τ, E) .

Theorem 3.6 *Let (X, τ, E) be a soft A_1-space. Then,*

a) (G, E) is a soft open set in $(X, \tau, E) \Leftrightarrow$ If the soft sequence $\left\{x_{e_n}^n\right\}$ converges to soft point $x_{e_0}^0 \in (G, E)$, then there exists $n_0 \in N$ with $\forall\, n \geq n_0$, $x_{e_n}^n \in (G, E)$.

b) (F, E) is a soft closed set in $(X, \tau, E) \Leftrightarrow$ If $\left\{x_{e_n}^n\right\} \subset (F, E)$ and $\left\{x_{e_n}^n\right\}$ converges to $x_{e_0}^0$, then $x_{e_0}^0 \in (F, E)$.

Proof. a) Sufficiency: It is obvious.

Necessity: Suppose that (G, E) is not a soft open set. Then soft point $x_{e_0}^0$ can be found but it is not soft interior point of (G, E) . If the family

$$\{(V_n, E) : (V_{n+1}, E) \subset (V_n, E)\}$$

is a soft neighborhood base of $x_{e_0}^0$, then $(V_n, E) \nsubseteq (G, E)$ for all $n \in \mathbb{N}$. Thus there exists soft point $x_{e_n}^n \in (V_n, E)$ such that $x_{e_n}^n \notin (G, E)$, for all $n \in \mathbb{N}$. Then the soft sequence $\left\{x_{e_n}^n\right\}$ converges to $x_{e_0}^0$. From the condition of theorem, there exists $n_0 \in \mathbb{N}$ such that $x_{e_n}^n \in (G, E)$ for all $n \geq n_0$. This is a contradiction. Hence (G, E) is a soft open set in (X, τ, E) .

b) Sufficiency: Let $\left\{x_{e_n}^n\right\} \subset (F, E)$ and $\left\{x_{e_n}^n\right\}$ converges to $x_{e_0}^0$. Suppose that $x_{e_0}^0 \notin (F, E)$. Thus $x_{e_0}^0 \in (F, E)^c$ .Then there exists $n_0 \in \mathbb{N}$ such that $x_{e_n}^n \in (F, E)^c$, for all $n \geq n_0$. This is a contradiction. Hence $x_{e_0}^0 \in (F, E)$.

Necessity: We want to show that $(F, E) = \overline{(F, E)}$. We consider soft neighborhood base as $\{(V_n, E) : (V_{n+1}, E) \subset (V_n, E)\}$ of soft point $x_e \in \overline{(F, E)}$. Then for all $n \in \mathbb{N}, (V_n, E) \cap (F, E) \neq \Phi$. Thus $x_{e_n}^n \in (V_n, E) \cap (F, E)$, for all $n \in \mathbb{N}$, we obtain soft sequence $\left\{x_{e_n}^n\right\} \subset (F, E)$. It is clear that $\left\{x_{e_n}^n\right\}$ converges to x_e. From the condition of theorem, $x_e \in (F, E)$. This implies that $\overline{(F, E)} \subset (F, E)$. Hence $(F, E) = \overline{(F, E)}$ is obtained.

Definition 3.6 *Let (X,τ,E) and (Y,τ',E') be two soft topological spaces, $(f,\varphi):(X,\tau,E)\to(Y,\tau',E')$ be a soft mapping and $x^0_{e_0}\in(X,\tau,E)$ be a soft point. The soft mapping (f,φ) is called a soft sequentially continuous at $x^0_{e_0}$ if each soft sequence $\left\{x^n_{e_n}\right\}$ converges to $x^0_{e_0}$ in (X,τ,E) implies $(f,\varphi)\left(\left\{x^n_{e_n}\right\}\right)$ converges to $(f,\varphi)\left(x^0_{e_0}\right)$ in (Y,τ',E').*

Proposition 3.3 *Let (X,τ,E) and (Y,τ',E') be two soft topological spaces. If $(f,\varphi):(X,\tau,E)\to(Y,\tau',E')$ is a soft sequentially continuous, then $f:(X,\tau_e)\to(Y,\tau'_{\varphi(e)})$ is a sequentially continuous, for each $e\in E$.*

Proof. Suppose that the sequence $\{x^n\}$ converges to x^0 in (X,τ_e). From the Remark 2, the soft sequence $\{x^n_e\}$ converges to soft point x^0_e in (X,τ,E). Since $(f,\varphi):(X,\tau,E)\to(Y,\tau',E')$ is a soft sequentially continuous, then $(f,\varphi)(x^n_e)=(f(x^n))_{\varphi(e)}$ converges to soft point $\left(f\left(x^0\right)\right)_{\varphi(e)}$. Hence $f(x^n)$ converges to $f\left(x^0\right)$.

Theorem 3.7 *Let (X,τ,E) and (Y,τ',E') be two soft topological spaces. If $(f,\varphi):(X,\tau,E)\to(Y,\tau',E')$ is a soft continuous, then it is also soft sequentially continuous.*

Proof. Suppose that $(f,\varphi):(X,\tau,E)\to(Y,\tau',E')$ is a soft continuous at x^0_e and the soft sequence $\left\{x^n_{e_n}\right\}$ converges to soft point $x^0_{e_0}$ in (X,τ,E). Since $(f,\varphi):(X,\tau,E)\to(Y,\tau',E')$ is a soft continuous, there exists a soft neighborhood $(U,E)\in\bar{U}\left(x^0_{e_0}\right)$ such that $(f,\varphi)((U,E))\subset(V,E')$, for each soft neighborhood (V,E') of $\left(f\left(x^0\right)\right)_{\varphi(e_0)}$. Since the soft sequence $\left\{x^n_{e_n}\right\}$ converges to soft point $x^0_{e_0}\in(U,E)$, then there exists $n_0\in\mathbb{N}$ such that $x^n_{e_n}\in(U,E)$, for all $n\geq n_0$. Hence $(f,\varphi)\left(x^n_{e_n}\right)\in(V,E')$ is obtained, for all $n\geq n_0$. This implies that (f,φ) is a soft sequentially continuous.

Theorem 3.8 *Let (X,τ,E) be a soft A_1-space. If $(f,\varphi):(X,\tau,E)\to(Y,\tau',E')$ is a soft sequentially continuous, then it is also soft continuous.*

Proof. Suppose that the soft sequentially continuous $(f,\varphi):(X,\tau,E)\to(Y,\tau',E')$ is not a soft continuous mapping. Then there exists a soft neighborhood $(V,E')\in\bar{U}\left((f,\varphi)\left(x^0_{e_0}\right)\right)$ such that $(f,\varphi)((U,E))\not\subseteq(V,E')$, for each soft neighborhood $(U,E)\in\bar{U}\left(x^0_{e_0}\right)$. Then $(f,\varphi)((V_n,E))\not\subseteq(V,E')$ is obtained, for soft neighborhood base

$$\{(V_n,E):(V_{n+1},E)\subset(V_n,E)\}$$

of $x^0_{e_0}$. Thus exists a soft point $x^n_{e_n}\in(V_n,E)$ such that $(f,\varphi)\left(x^n_{e_n}\right)\not\subseteq(V,E')$, for all $n\in\mathbb{N}$. The soft sequence $\left\{x^n_{e_n}\right\}$ converges to $x^0_{e_0}$ in (X,τ,E), but $(f,\varphi)\left(\left\{x^n_{e_n}\right\}\right)$ does not converge to $(f,\varphi)\left(x^0_{e_0}\right)$ in (Y,τ',E').

Corollary 3.1 *We give some new concepts in soft topological spaces such as soft first-countable spaces, soft second-countable spaces and soft separable spaces, soft sequential continuity and some important properties of these concepts are investigated.*

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