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On strongly uniformly paracompact spaces

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Abstract. *In this paper we present a new approach to the definition of strong uniform paracompactness of uniform and ordered uniform spaces. Their connection with other properties of compactness type of uniform spaces is studied, an internal characterization is given, the characterization of this property of uniform spaces are established by means of finitely additive open covers, mappings and Hausdorff compact extensions, and the characterization of compactness of topological spaces is established through uniform structures.*

Keywords. Uniformly σ-star finite open cover, σ-locally finite uniform cover, finitely additive open cover, strongly uniformly paracompact, strongly paracompact, uniformly paracompact, ordered uniform space.

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1 Introduction

Throughout this paper, all uniform spaces are assumed to be Hausdorff, topological spaces are Tychonoff, and mappings are uniformly continuous.

It is well known that paracompactness play an important role in General Topology. Therefore, the finding of uniform analogues of paracompactness and strong paracompactness is an interesting problem in the theory of uniform spaces. Many mathematicians turned

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to this problem and as a result different variants of uniform paracompactness and strong uniform paracompactness of uniform spaces appeared. For example, uniform R-paracompactness in the sense of M.D. Rice [1], uniform F-paracompactness in the sense of Z. Frolik [2], uniform B-paracompactness in the sense of A.A. Borubaev [3], uniform P-paracompactness in the sense of B.A. Pasynkov [4], uniform A-paracompactness in the sense of L.V. Aparina [5], uniform I-paracompactness in the sense of J.R. Isbell [6], strong uniform R-paracompactness in the sense of D.K. Musaev [7], strong uniform K-paracompactness in the sense of B.E. Kanetov [8].

For a cover α of a set X and $x \in X$, $M \subset X$, we have: $St(\alpha, x) = \{A \in \alpha : A \ni x\}$ $\alpha(x) = \bigcup St(\alpha, x)$, $St(\alpha, M) = \{A \in \alpha : A \bigcap M \neq \varnothing\}$, $\alpha(M) = \bigcup St(\alpha, M)$. For covers α and β of the set X, the symbol $\alpha \succ \beta$ means that the cover α is a refinement of the cover β, i.e. for any $A \in \alpha$ there exists $B \in \beta$ such that $A \subset B$ and, for covers α and β of a set X, we have: $\alpha \wedge \beta = \{A \bigcap B : A \in \alpha, B \in \beta\}$. For a cover α of a set X let $\alpha^{\angle} = \{\bigcup \alpha_0 : \alpha_0 \subset \alpha$ - is finite$\}$. The cover α is finitely additive if $\alpha^{\angle} = \alpha$. The cover consisting of the union of countable number of (uniformly) locally finite families is called (uniformly) σ-locally finite. For the uniformity U by τ_U we denote the topology generated by the uniformity and symbol U_X means the universal uniformity.

2 Strong uniform paracompactness

Star finitely uniformly σ-locally finite covers will be called uniformly σ-star finite.

Definition 2.1 *A uniform space (X, U) is said to be uniformly paracompact, if every open cover of (X, U) has a uniformly σ-star finite open refinement.*

Proposition 2.1 *If (X, U) is a strongly uniformly paracompact space, then the topological space (X, τ_U) is strongly paracompact. Conversely, if (X, τ) is strongly paracompact, then the uniform space (X, U_X) is strongly uniformly paracompact, where U_X is the universal uniformity of the space X.*

Proof. Let α be an arbitrary open cover of the space (X, τ_U). Then there exist a uniformly star finite open cover β such that $\beta \succ \alpha$. Since any uniformly star finite open cover is a star finite open cover, then the cover β is a star finite. Thus the space (X, τ_U) is strongly paracompact. Conversely, let (X, τ) be a strongly paracompact space. Then the set of all open covers forms a base of the universal uniformity U_X of the strongly paracompact space (X, τ). It is easy to see that the uniform space (X, U_X) is a strongly uniformly paracompact space.

Recall [4] that a uniform space (X, U) is called uniformly paracompact, if every open cover of X has a uniformly σ-locally finite open refinement.

Proposition 2.2 *Any strongly uniformly paracompact space (X, U) is uniformly paracompact.*

The proof follows easily from the definition of strong uniform paracompactness.

Theorem 2.1 *For a uniform space (X, U) the following conditions are equivalent:*

1) (X, U) *is strongly uniformly paracompact;*
2) (X, U) *is uniformly paracompact and the topological space (X, τ_U) is strongly paracompact.*

Proof. 1. $\Rightarrow$ 2. Obviously.

2. $\Rightarrow$ 1. Let λ be an arbitrary open cover of the space (X, U). Then there exists such star finite open cover α of the space (X, τ_U), that $\alpha \succ \lambda$. Since the uniform space (X, U)

is uniformly paracompact, then in the cover α we inscribe a uniformly σ-locally finitely open cover $\beta = \{\beta_i : i \in N\}$ of the space (X, U). For each $B_{\beta_i} \in \beta_i$ pick $A_{B_{\beta_i}} \in \alpha$, such that $B_{\beta_i} \subset A_{B_{\beta_i}}$. Put $\alpha_i = \{A_{B_{\beta_i}} : B_{\beta_i} \in \beta_i\}$. It's clear that $\beta_i \succ \alpha_i$. Then $\alpha_0 = \{\alpha_i : i \in N\}$ is an open cover that is a countable union of subfamilies of (X, U). It is easy to see that the latter is a star finite cover. We show that α_0 is a uniformly σ-locally finite cover. For any $i \in N$ there is such a uniform cover $\gamma_{\beta_i} \in U$, that $|St(\beta_i, \Gamma_{\beta_i})| < \aleph_0$ for any $\Gamma_{\beta_i} \in \gamma_{\beta_i}$. Let $B_{\beta_i} \in St(\beta_i, \Gamma_{\beta_i})$. Since $\beta_i \succ \alpha_i$, then for any $B_{\beta_i} \in \beta_i$ there is $A_{B_{\beta_i}} \in \alpha_i$, such that $B_{\beta_i} \subset A_{B_{\beta_i}}$. By virtue of the star finiteness of the cover α_0 we have that $|St(\alpha_0, A_{\beta_i})| < \aleph_0$. It follows that $|St(\alpha_0, B_{\beta_i})| < \aleph_0$ for any $B_{\beta_i} \in St(\beta_i, \Gamma_{\beta_i})$. Then $|St(\alpha_0, \beta_i(\Gamma_{\beta_i}))| < \aleph_0$, i.e. $|St(\alpha_0, \Gamma_{\beta_i})| < \aleph_0$ for any $\Gamma_{\beta_i} \in \gamma_{\beta_i}$. Consequently, the subfamily α_i is uniformly locally finite. Hence, the cover α_0 is a star finite and uniformly σ-locally finite cover i.e. uniformly σ -star finite open cover which refines λ. Thus, the uniform space (X, U) is strongly uniformly paracompact.

Corollary 2.1 *Any uniformly paracompact space (X, U), whose topological space (X, τ_U) is locally compact is strongly uniformly paracompact.*

Corollary 2.2 *Any compact space is strongly uniformly paracompact.*

Recall [3] that a uniform space (X, U) is said to be uniformly locally compact if there exists a uniform cover consisting of compact subsets. A uniform space (X, U) is said to be uniformly R-paracompact, if every open cover of X has a uniformly locally finite open refinement [1].

The following theorem is a uniform analogue of A.V. Arhangel'skii's theorem that every locally compact topological group is strongly paracompact [10].

Theorem 2.2 *Any uniformly locally compact space (X, U) is strongly uniformly paracompact.*

Proof. Any uniformly locally compact space (X, U) is uniformly paracompact, and its topological space (X, τ_U) is strongly paracompact. Then, according to Theorem 1, the uniform space (X, U) is strongly uniformly paracompact.

Corollary 2.3 *Any uniformly locally compact space(X, U) is strongly uniformly R-paracompact.*

Recall [9], that a uniform space (X, U) is said to be strongly uniformly $B(= P)$-paracompact if it is uniformly $B(= P)$-paracompact ([3], [4]) and its topological space (X, τ_U) is strongly paracompact.

Corollary 2.4 *Any strongly uniformly $B(= P)$-paracompact space (X, U) is strongly uniformly paracompact.*

Proposition 2.3 *Every separable metrizable uniform space (X, U) is strongly uniformly paracompact.*

Proof. Since every separable metrizable uniform space (X, U) is strongly uniformly $B(= P)$-paracompact, than according to Corollary 4 the uniform space (X, U) is strongly uniformly paracompact.

Corollary 2.5 *Any strongly uniformly R-paracompact space (X, U) is strongly uniformly paracompact.*

It is clear that noncomplete separable metrizable spaces are not strongly uniformly R-paracompact.

The following theorem is a characterization of strongly uniformly paracompact spaces, with the help of finitely additive open covers.

Theorem 2.3 *For a uniform space (X, U) the following conditions are equivalent:*

1) (X, U) *is strongly uniformly paracompact;*
2) *Every finitely additive open cover of (X, U) has a uniformly σ-star finite open refinement.*

Proof. 1. $\Rightarrow$ 2. Let (X, U) be a strongly uniformly paracompact space and α be an arbitrary finitely additive open cover. Then there is a uniformly σ-star open refinement β of α. Consequently, the space (X, U) is strongly uniformly paracompact.

2. $\Rightarrow$ 1. Suppose any uniformly σ-star finite open cover of space (X, U) can be refined by finitely additive open cover of the space (X, U). We show that (X, U) is a strongly uniformly paracompact space. Let α be an arbitrary open cover. By virtue of strong uniform paracompactness of (X, U) in a finitely additive open cover $\alpha^{\angle}$ we will inscribe a uniformly σ-star finite open cover β of (X, U). Then for any i and $B \in \beta_i$ there exist $A_B^{\angle} \in \alpha^{\angle}$ such that $B \subset A_B^{\angle}$, $A_B^{\angle} = \bigcup\limits_{k=1}^{n} A_k$, $A_k \in \alpha$. Put $\alpha_0 = \bigcup\{\alpha_{\beta_i} : i \in N\}$, where $\alpha_{\beta_i} = \{A_B^{\angle} : B \in \beta_i\}$. It is easy to see that α_0 is a uniformly σ-star finite open cover and $\alpha_0 \succ \alpha$. Consequently, the space (X, U) is strongly uniformly paracompact.

As a consequence of Theorem 2.3., we obtain Theorem 3.4. (see [22], p. 120) on strongly uniformly R-paracompact spaces.

Corollary 2.6 *For a uniform space (X, U) the following conditions are equivalent:*

1) (X, U) *is strongly uniformly R-paracompact;*
2) *Every finitely additive open cover of the space (X, U) has a uniformly star finite open refinement.*

Theorem 2.4 *Let (X, U) be a uniform space and cX be certain compact Hausdorff extensions of the space (X, τ_U). Then the following conditions are equivalent:*

1) (X, U) *is strongly uniformly paracompact;*
2) *For each compact $K \subset cX\backslash X$ there exists a uniformly σ-star finite open cover α of (X, U) such that $cl_{cX} A \bigcap K = \varnothing$ for all $A \in \alpha$.*

Proof. 1. $\Rightarrow$ 2.Let (X, U) be a strongly uniformly paracompact space and $K \subset cX\backslash X$ an arbitrary compact set. Then for each point $x \in X$ there is such an open in cX neighborhood O_x such that $cl_{cX} O_x \bigcap K = \varnothing$. Put $\beta = \{O_x \bigcap X : x \in X\}$. It is clear that the latter is an open cover of the space (X, U). Since the uniform space (X, U) is strongly uniformly paracompact, then there is a uniformly σ-star finite open cover γ which refines β. Then $cl_{AX}\varGamma \subset cl_{cX}(\bigcup\limits_{i=1}^{n}(O_{x_i} \bigcap X)) \subset \bigcup\limits_{i=1}^{n} cl_{cX} O_{x_i}$. From here we obtain $cl_{cX}\varGamma \bigcap K = \varnothing$ for any $\varGamma \in \gamma$.

2. $\Rightarrow$ 1. Let α be an arbitrary finitely additive open cover of the space (X, U). Then there exists an open family γ in cX such that $\gamma \wedge \{X\} = \alpha$. Let $K = cX\backslash \bigcup \gamma$. Then K is compact, therefore there is such a uniform σ-star finite open cover β that $cl_{cX} B \bigcap K = \varnothing$ for any $B \in \beta$. The compact $cl_{cX} B$ lies in cX, therefore there exist $\varGamma_1, \varGamma_2, ..., \varGamma_n \in \gamma$ such that $cl_{cX} B \subset \bigcup\limits_{i=1}^{n} \varGamma_i$. Then $B \subset \bigcup\limits_{i=1}^{n} A_i$, where $\bigcup\limits_{i=1}^{n} A_i \in \alpha$. Consequently, (X, U) is strongly uniformly paracompact.

This theorem is a uniform analogue of H. Tamano's theorem for strongly paracompact spaces.

Corollary 2.7 *Let (X, U) be a uniform space, cX be some compactification of the space (X, τ_U). The uniform space (X, U) is strongly uniformly R-paracompact if and only if for any compact $K \subset cX \backslash X$ there exists a uniformly star finite open cover α such that $cl_{cX} A \bigcap K = \varnothing$ for any $A \in \alpha$.*

The following theorem shows the equivalence of various variants of strong uniform paracompactness in the class of $\aleph_0$-bounded uniform spaces.

Proposition 2.4 *For an $\aleph_0$-bounded uniform space (X, U) the following conditions are equivalent:*

1) *(X, U) is strongly uniformly P-paracompact;*
2) *(X, U) is strongly uniformly B-paracompact;*
3) *(X, U) is strongly uniformly paracompact;*
4) *The topological space (X, τ_U) Lindelöf.*

The proof follows from the following Lemmas 2.1 and 2.2.

Lemma 2.1 *If for a uniform space (X, U) its topological space (X, τ_U) is Lindelöf, then the uniform space (X, U) is strongly uniformly P-paracompact.*

Proof. Since space (X, τ_U) is Lindelöf, it is strongly paracompact. According to Lemma 2.10 ([4]) the uniform space (X, U) is uniformly paracompact. Consequently, the uniform space (X, U) is strongly uniformly P-paracompact.

Lemma 2.2 *If a uniform space (X, U) is strongly uniformly paracompact and $\aleph_0$-bounded, then the topological space (X, τ_U) is a Lindelöf.*

Proof. Let λ be an open cover of the space (X, τ_U). Then there exists a uniformly σ-star finite open cover $\alpha = \{\alpha_i : i \in N\}$ inscribed in λ. Since the $\alpha = \{\alpha_i : i \in N\}$ is star finite and uniformly σ-locally finite, then for each $i \in N$ there is $\beta_i \in U$ such that each element of which intersects only a finite number of elements of the family α_i. For each member $i \in N$ of the family α_i has a non-empty intersection with some elements of the covers β_i. It follows that families α_i are countable, i.e. cover $\alpha = \{\alpha_i : i \in N\}$ is countable. Therefore, the topological space (X, τ_U) is Lindelöf.

Corollary 2.8 *For a Tychonoff space the following properties are equivalent:*

1) *The topological space (X, τ) is Lindelöf;*
2) *For each uniformity U such that $\tau_U = \tau$ the uniform space (X, U) is strongly uniformly P-paracompact;*
3) *For each uniformity U such that $\tau_U = \tau$ the uniform space (X, U) is strongly uniformly B-paracompact;*
4) *For each uniformity U such that $\tau_U = \tau$ the uniform space (X, U) is strongly uniformly paracompact.*

A continuous mapping f of a topological space X onto a topological space Y is said to be perfect if f closed and the inverse images under f of points $y \in Y$ are compact subspaces of X. A continuous mapping f of a topological space X to a topological space Y is said to be ω-mapping if for each point $y \in Y$ there exist such neighborhood O_y and $W \in \omega$, that $f^{-1}(O_y) \subset W$.

The following statement describes the preservation of strong paracompactness in the preimage direction under perfect mappings.

Theorem 2.5 *Let $f : (X, U) \to (Y, V)$ be a perfect uniformly continuous mapping of a uniform space (X, U) onto a uniform space (Y, V). If (Y, V) is strongly uniformly paracompact, then (X, U) is strongly uniformly paracompact.*

Proof. Let ω be a finite additive open cover of the space (X,U). Then it is easy to see that mapping $f:(X,U)\to(Y,V)$ is ω-mapping. Then for each point $y\in Y$ there exists a neighborhood O_y whose preimage is contained in some element of the cover ω. Put $\beta=\{O_y : y\in Y\}$. Since (Y,V) is strongly uniformly paracompact, then there exists a uniformly σ-star finite open cover α is refinement of the cover β. Then it is easy to see that a uniform σ-star finite open cover $f^{-1}\alpha$ is refinement of ω. Therefore, the uniform space (X,U) is strongly uniformly paracompact.

A uniform space (X,U) is called μ-complete if every filter Cauchy having a base of cardinality $\leq\mu$ converges in it.

Lemma 2.3 *A Tychonoff space X is μ-compact if and only if for each uniformity U on X, the uniform space (X,U) is μ-complete.*

Proof. Let F be a Cauchy filter on the uniform space (X,U) having a base of cardinality $\leq\mu$. Than F has a limit points in the uniform space (X,U). Therefore, (X,U) is μ-complete. Conversely, let F be a ultrafilter, which has a base of cardinality $\leq\mu$. Let (X,U) be a uniform space such that $\tau_U=\tau$. Then there exists a precompact uniformity U_P such that $\tau_{U_P}=\tau_U=\tau$. Clearly, F is a Cauchy filter in the precompact uniform space (X,U_P). Hence ultrafilter F converges to some points in (X,U_P), i.e. (X,τ) is μ-compact.

Theorem 2.6 *For a Tychonoff space (X,τ) the following conditions are equivalent:*

1) (X,τ) *is compact;*
2) (X,U) *is $\aleph_0$-complete and strongly uniformly paracompact.*

The proof follows from Lemma 2.3 and Corollary 2.8.

A uniform space (X,U) is said to be uniformly paracompact, if every finitely additive open cover of X has a σ-locally finite uniform refinement.

If a uniform space (X,U) is uniform paracompact then the topological space (X,τ) is paracompact. Conversely, if (X,τ) is a paracompact space then the uniform space (X,U_X) is uniform paracompact, where U_X is the universal uniformity of the space X.

Theorem 2.7 *Let (X,U) be a uniform space and cXbe certain compact Hausdorff extensions of the space (X,τ_U). Then the following conditions are equivalent:*

1) (X,U) *is uniformly paracompact;*
2) *For each compact $K\subset cX\backslash X$ there exists a σ-star finite uniformly cover $\alpha\in U$ such that $cl_{cX}A\bigcap K=\varnothing$ for all $A\in\alpha$.*

Theorem 2.8 *Let $f:(X,U)\to(Y,V)$ be a perfect uniformly continuous mapping of a uniform space (X,U) onto a uniform space (Y,V). If (Y,V) is uniformly paracompact, then (X,U) is uniformly paracompact.*

A uniform space (X,U) is said to be strongly uniformly locally compact if there exists a σ-locally finite uniform cover consisting of compact subsets.

Every strongly uniformly locally compact space is uniformly paracompact and every uniformly paracompact space is uniformly R-paracompact. A non complete metrizable uniform space is not uniformly paracompact.

A triple $(X,<,U)$ is called an ordered uniform space if:

1) The uniformity U generates on X interval topology;
2) The uniformity U has a base B, each cover of which consists of intervals [3].

In this case the uniformity U is called an ordered uniformity of the ordered space $(X,<)$.

Ordered topological space is understood as a linearly ordered set, endowed with the interval topology, and the interval is any convex subset which is open in the interval topology.

If the cover α of an ordered space $(X,<)$ consists of intervals, then cover α is called interval.

Theorem 2.9 *For an ordered uniform space $(X, <, U)$ the following conditions are equivalent:*

1) $(X, <, U)$ *is strongly uniformly paracompact;*
2) $(X, <, U)$ *is uniformly paracompact.*

Proof. $1) \Rightarrow 2)$ Obviously.

$2) \Rightarrow 1)$. Let $(X, <, U)$ be uniformly paracompact ordered uniform space. Then the ordered topological space $(X, <, \tau_U)$ is paracompact. According to Fedorchuk's theorem (see. [13]) it is strongly paracompact. Then according to Theorem 2.1, the ordered uniform space $(X, <, U)$ is strongly uniformly paracompact.

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