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MATHEMATICS

Absolute Convergence Of Spectral Expansion Of Absolutely Continuous Vector-Function In Eigen Vector-Functions Of Third Order Differential Operator

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Abstract. *In the paper we consider an ordinary differential operator of third order with matrix coefficients, study absolute and uniform convergence of expansion of an absolutely continuous vector-function in eigen vector-functions of the given operator and estimate the residual of this expansion.*

Keywords. Vector-functions, differential operator.

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Let us consider on the interval $G = (0, 1)$ the operator

$$L\Psi = \Psi^{(3)} + U_2(x)\Psi^{(1)} + U_3(x)\Psi$$

with matrix coefficients $U_l(x) = \left(u_{lij}(x)\right)_{i,j=1}^{m}$, $l = 2, 3$, where $u_{lij}(x) \in L_1(G)$.

Denote by $D(G)$ the class of m-component vector-functions absolutely continuous together with own derivatives to second order, inclusively on the interval $\overline{G} = [0, 1]$ $\left(D(G) = W_{1,m}^3(G)\right)$.

Under the eigen vector-function of the operator L responding to the eigen value λ we understand any identically not equal to zero vector-function $\Psi(x) = (\Psi_1(x), \Psi_2(x), ..., \Psi_m(x))^T \in D(G)$ satisfying almost every where in G the equation (see [1])

$$L\Psi + \lambda\Psi = 0.$$

Let $L_p^m(G), p \geq 1$ be a space of m-component vector-functions $f(x) = (f_1(x), f_2(x), ..., f_m(x))^T$ with the norm

$$\|f\|_{p,m} = \left\{\int_G |f(x)|^p\, dx\right\}^{1/p} = \left\{\int_G \left(\sum_{l=1}^{m} |f_l(x)|^2\right)^{p/2} dx\right\}^{1/p}.$$

Suppose that $\{\Psi_k(x)\}_{k=1}^{\infty}$ is a complete orthonormed in $L_2^m(G)$ system consisting of vector-functions of the operator L. Denote by $\{\lambda_k\}_{k=1}^{\infty}$ the appropriate system of eigen values $(\mathrm{Re}\lambda_k = 0)$.

Denote $\mu_k = \begin{cases} (-i\lambda_k)^{1/3}, & if \quad \mathrm{Im}\lambda_k \geq 0, \\ (i\lambda_k)^{1/3}, & if \quad \mathrm{Im}\lambda_k < 0, \end{cases}$ and consider a partial sum of orthogonal expansion of the vector-function $f(x) \in W_{1,m}^1(G)$ in the system $\{\Psi_k(x)\}_{k=1}^{\infty}$

$$\sigma_\nu(x, f) = \sum_{\mu_k \leq \nu} f_k \Psi_k(x), \quad \nu > 0,$$

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where

$$f_k = (f, \Psi_k) = \int_0^1 < f(x), \Psi_k(x) > dx = \int_0^1 \sum_{l=1}^{m} f_l(x) \overline{\Psi_{kl}}(x) dx,$$

$$\Psi_k(x) = (\Psi_{k1}(x), \Psi_{k2}(x), ..., \Psi_{km}(m)).$$

In the paper we prove the following theorem.

Theorem. *Let the vector-function $f(x)$ belong to the class $W^1_{1,m}(G)$, the system $\{\Psi_k(x)\}_{k=1}^{\infty}$ be uniformly bounded, and the following conditions be fulfilled*

$$\left| < f,(x), \Psi_k^{(2)}(x) > |_0^1 \right| \le C(f) \mu_k^{\alpha}, \quad 0 \le \alpha < 2, \mu_k \ge 4\pi \tag{1}$$

$$\sum_{n=2}^{\infty} n^{-1} \omega_{1,m} \left(f', n^{-1} \right) < \infty. \tag{2}$$

Then the expansion of the vector-function $f(x)$ in the system $\{\Psi_k(x)\}_{k=1}^{\infty}$ converges absolutely and uniformly on $\overline{G} = [0,1]$, and the following estimation is valid

$$\sup_{x \in \overline{G}} |\sigma_\nu(x,f) - f(x)| \le const \left\{ C(f) \nu^{\alpha-2} + \sum_{n=[\nu]}^{\infty} n^{-1} \omega_{1,m} \left(f, n^{-1} \right) \right.$$

$$\left. + \nu^{-1} \left\| f' \right\|_{1,m} + \left(\|f\|_{\infty,m} + \left\| f' \right\|_{1,m} \right) \sum_{r=2}^{3} \nu^{1-r} \| \| U_r \| \|_1 \right\}, \tag{3}$$

where $\omega_{1,m}(g,\delta)$ is an integral modulus of continuity of the vector-function $g(x) = (g_1(x), g_2(x), ..., g_m(x))^T \in L_1^m(G)$; $\|\|U_r\|\|_1 = \sum_{i,j=1}^{m} \left\| u_{rij} \right\|_1$, $r = 2,3$; const is independent of $f(x)$.

Note that such theorems for a second order operator were proved in the papers [2]-[5].

For proving the theorem, it is necessary to estimate the Fourier coefficient f_k of the vector-function $f(x) \in W^1_{1,m}(G)$.

Lemma. *Let the system $\{\Psi_k(x)\}_{k=1}^{\infty}$ be uniformly bounded. Then for the Fourier coefficients f_k of the vector-function $f(x) \in W^1_{1,m}(G)$ satisfying condition (1), the estimation ($\mu_k \ge 4\pi$) is valid*

$$|f_k| \le const \left\{ C(f) \mu_k^{\alpha-3} + \mu_k^{-1} \omega_{1,m} \left(f, \mu_k^{-1} \right) + \mu_k^{-2} \left\| f'_{1,m} \right\|_{1,m} \right.$$

$$\left. + \mu_k^{-2} \left(\|f\|_{\infty,m} + \left\| f' \right\|_{1,m} \right) \sum_{r=2}^{3} \mu_k^{2-r} \| \| U_r \| \|_1 \right\}. \tag{4}$$

The proof of the lemma. For the eigen vector-function $\Psi_k(x)$ the following formulas are valid:

$$\mu_k^{-l} \Psi_k^{(l)}(t) = \sum_{j=1}^{2} X_j^-(0) \left(-i\omega_j \right)^l e^{-i\omega_j \mu_k t} + \left(-i\omega_j \right)^l B_3^- e^{i\omega_3 \mu_k (1-t)}$$

$$- \sum_{j=1}^{2} (-i)^l \omega_j^{l+1} \int_0^t M(\Psi_k(\xi)) e^{i\omega_j \mu_k (\xi - t)} d\xi$$

$$+ (-i)^l \omega_j^{l+1} \int_t^1 M(\Psi_k(\xi)) e^{i\omega_3 \mu_k (\xi - t)} d\xi, \tag{5}$$

for $\mathrm{Im}\lambda_k > 0$;

$$\mu_k^{-l}\Psi_k^{(l)}(t) = \sum_{j=1,j\neq 2}^{3} \left(i\omega_j\right)^l X_j^{+}(0)\, e^{i\omega_j\mu_k t} + (i\omega_2)^l B_2^{+} e^{-i\omega_2\mu_k(1-t)}$$

$$-\sum_{j=1,j\neq 2}^{3} (i)^l \omega_j^{l+1} \int_0^t M\left(\Psi_k(\xi)\right) e^{-i\omega_j\mu_k(\xi-t)} d\xi$$

$$+(i)^l \omega_2^{l+1} \int_t^1 M\left(\Psi_k(\xi)\right) e^{-i\omega_2\mu_k(\xi-t)} d\xi; \tag{6}$$

for $\mathrm{Im}\lambda_k < 0$. Moreover, $l = \overline{0,2}$; $\omega_1 = -1$, $\omega_2 = e^{-i\pi/3}$, $\omega_3 = e^{i\pi/3}$;

$$B_3^{-} = X_3^{-}(0)\, e^{-i\omega_3\mu_k} - \omega_3 \int_0^1 M\left(\Psi_k(\xi)\right) e^{-i\omega_3\mu_k(\xi-1)} d\xi;$$

$$B_2^{+} = X_2^{+}(0)\, e^{i\omega_2\mu_k} - \omega_2 \int_0^1 M\left(\Psi_k(\xi)\right) e^{i\omega_2\mu_k(\xi-1)} d\xi;$$

$$X_j^{\pm}(x) = \frac{1}{3\mu_k^2} \sum_{m=0}^{2} (\mp i\mu_k)^m \omega_j^{m+1} \Psi_k^{(2-m)}(x);$$

$$M\left(\Psi_k(\xi)\right) = \frac{1}{3\mu_k^2} \sum_{r=2}^{3} U_r(\xi)\, \Psi_k^{(3-r)}(\xi).$$

Subject to definition of the eigen function $\Psi_k(x)$ calculate the Fourier coefficients f_k for $\mu_k \geq 1$:

$$f_k = (f, \Psi_k) = \left(f, -\lambda_k^{-1} L\Psi_k\right) = -\overline{\lambda_k^{-1}}\,(f_k, L\Psi_k) = -\overline{\lambda_k^{-1}} \left(f, \Psi_k^{(3)}\right)$$

$$-\overline{\lambda_k^{-1}} \left(f, U_2\Psi_k^{(1)}\right) - \overline{\lambda_k^{-1}}\,(f, U_3\Psi_k). \tag{7}$$

Taking into account estimations (see [6])

$$\left\|\Psi_k^{(s)}\right\|_{\infty,m} \leq const\,(1+|\mu_k|)^{s+\frac{1}{p}} \|\Psi_k\|_{m,p}, \;\; p \geq 1, \tag{8}$$

and uniform boundedness of the system $\{\Psi_k(x)\}_{k=1}^{\infty}$, we find

$$\frac{1}{|\lambda_k|} \left|\left(f, U_2\Psi_k^{(1)}\right)\right| \leq \frac{const}{\mu_k^3} \|\|U_2\|\|_1 \|f\|_{\infty,m} \left\|\Psi_k^{(1)}\right\|_{\infty,m}$$

$$\leq \frac{const}{\mu_k^2} \|\|U_2\|\|_1 \|f\|_{\infty,m}; \tag{9}$$

$$\frac{1}{|\lambda_k|} |(f, U_3\Psi_k)| \leq \frac{const}{\mu_k^3} \|\|U_3\|\|_1 \|f\|_{\infty,m}.$$

For estimating the first summand in the right side of equality (7), we make integration by parts and take into account condition (1):

$$\left|\frac{1}{\lambda_k}\right| \left|\left(f, \Psi_k^{(3)}\right)\right| \leq \frac{1}{\mu_k^3} \left|\left\langle f, \Psi_k^{(2)}\right\rangle |_0^1\right| + \frac{1}{\mu_k^3} \left|\left(f', \Psi_k^{(2)}\right)\right|$$

$$\leq C(f)\, \mu_k^{\alpha-3} + \frac{1}{\mu_k^3} \left|\left(f', \Psi_k^{(2)}\right)\right|. \tag{10}$$

For estimating the last summand, we use formula (5) or (6) depending on the sign $\mathrm{Im}\lambda_k$. For definiteness, we consider the case $\mathrm{Im}\lambda_k < 0$. Then by formula (6) for $l = 2$

$$\frac{1}{\mu_k^3}\left(f', \Psi_k^{(2)}\right) = \frac{1}{\mu_k}\left(f', \mu_k^{-2}\Psi_k^{(2)}\right) = \frac{1}{\mu_k}\sum_{j=1, j\neq 2}^{3}\overline{(i\omega_j)^2}\left(f', X_j^+(0)\, e^{i\omega_j\mu_k t}\right)$$

$$+\frac{1}{\mu_k}\overline{(i\omega_2)^2}\left(f', B_2^+ e^{-i\omega_2\mu_k(1-t)}\right)$$

$$-\frac{1}{\mu_k}\sum_{j=1, j\neq 2}^{3}\overline{i^2\omega_2^3}\left(f', \int_0^t M\left(\Psi_k(\xi)\right) e^{-i\omega_j\mu_k(\xi-t)}d\xi\right)$$

$$+\frac{1}{\mu_k}\overline{\omega_2^3}\left(f', \int_t^1 M\left(\Psi_k(\xi)\right) e^{-i\omega_2\mu_k(\xi-t)}d\xi\right). \tag{11}$$

Estimate each summand in the given equality. Passing to coordinates, we get

$$\overline{\left(f', X_j^+(0)\, e^{i\omega_j\mu_k t}\right)} = \sum_{l=1}^{m} X_{jl}^+(0)\int_0^1 \overline{f_l'(t)} e^{i\omega_j\mu_k t}dt, \quad j = 1, 3.$$

Here, taking into account estimation (8) for $p = \infty$, uniform boundedness of the system $\{\Psi_k(x)\}_{k=1}^{\infty}$ and estimation (see [7])

$$\left|\int_0^1 \overline{f_l'(t)} e^{i\omega_j\mu_k t}dt\right| \le const\left\{\omega_1\left(f_l', \mu_k^{-1}\right) + \mu_k^{-1}\left\|f_l'\right\|_1\right\}, \quad l = \overline{1, m}$$

we find

$$\left|\left(f', X_j(0)\, e^{i\omega_j\mu_k t}\right)\right| \le const\left\{\omega_{1,m}\left(f', \mu_k^{-1}\right) + \mu_k^{-1}\left\|f'\right\|_{1,m}\right\}, \quad j = 1, 3. \tag{12}$$

From formula (6) for $l = 0$ we get that for B_2^+ it is fulfilled

$$\left|B_2^+\right| \le const\left\|\exp\left(-i\omega_2\mu_k(1-\cdot)\right)\right\|_\infty^{-1}\left\{\left\|\Psi_k\right\|_{\infty,m}\right.$$

$$\left.+\sum_{j=1, j\neq 2}^{3}\left|X_j^+(0)\right| + \sum_{j=1, j\neq 2}^{3}\left\|\int_0^{\cdot}\left|M\left(\Psi_k(\xi)\right)\right|d\xi\right\|_\infty + \left\|\int_{\cdot}^1\left|M\left(\Psi_k(\xi)\right)\right|d\xi\right\|_\infty\right\}.$$

Here taking into account

$$\left|M\left(\Psi_k(\xi)\right)\right| \le \frac{1}{3\mu_k^2}\sum_{r=2}^{3}\left\|U_r(\xi)\right\|\left\|\Psi_k^{(3-r)}\right\|_{\infty,m} \le \frac{const}{\mu_k}\left[\sum_{r=2}^{3}\left\|U_r(\xi)\right\|\mu_k^{2-r}\right]$$

$$\left\|\Psi_k\right\|_{\infty,m} \le \frac{const}{\mu_k}\left[\sum_{r=2}^{3}\left\|U_r(\xi)\right\|\right]\mu_k^{2-r}; \tag{13}$$

$$\left|X_j^+(0)\right| \le const\left\|\Psi_k\right\|_{\infty,m} \le const$$

we find that for any $k \in N$ the following estimation is valid:

$$\left|B_2^+\right| \le const$$

Now, use this estimation in the second summand of equality (11) and get

$$\left|\frac{1}{\mu_k}\overline{(i\omega_2)^2}\left(f', B_2^+ e^{-i\omega_2\mu_k(1-t)}\right)\right| \le \frac{const}{\mu_k}\left\{\omega_{1,m}\left(f', \mu_k^{-1}\right) + \mu_k^{-1}\left\|f'\right\|_{1,m}\right\}. \tag{14}$$

For estimating the third and fourth summands in (11), we apply inequality (13) and have

$$\left|\left(f', \int_0^t M\left(\Psi_k\left(\xi\right)\right) e^{-i\omega_j \mu_k (\xi-t)} d\xi\right)\right|$$

$$\leq \frac{const}{\mu_k}\left(\sum_{r=2}^{3} \left\|\left\|U_r\right\|\right\|_1 \mu_k^{2-r}\right) \left\|f'\right\|_{1,m}, j=1,3 \tag{15}$$

$$\left|\left(f', \int_t^1 M\left(\Psi_k\left(\xi\right)\right) e^{-i\omega_2 \mu_k (\xi-t)} d\xi\right)\right| \leq \frac{const}{\mu_k}\left(\sum_{r=2}^{3} \left\|\left\|U_r\right\|\right\|_1 \mu_k^{2-r}\right) \left\|f'\right\|_{1,m}. \tag{16}$$

By (12), (14)-(16) from equality (11) it follows

$$\frac{1}{\mu_k^3}\left|\left(f', \Psi_k^{(2)}\right)\right|$$

$$\leq \frac{const}{\mu_k}\left\{\omega_{1,m}\left(f', \mu_k^{-1}\right) + \mu_k^{-1}\left\|f'\right\|_{1,m} + \mu_k^{-1}\left\|f'\right\|_{1,m} \sum_{r=2}^{3} \left\|\left\|U_r\right\|\right\|_1 \mu_k^{2-r}\right\}. \tag{17}$$

Taking into account estimations (9), (10) and (17) in equality (7), we get estimation (4) for the coefficient f_k. The lemma is proved.

The proof of the theorem. Represent the series $\sum_{k=1}^{\infty} |f_k| \left|\Psi_k\left(x\right)\right|$ in the form

$$\sum_{k=1}^{\infty} |f_k| \left|\Psi_k\left(x\right)\right| = \sum_{0\leq\mu_k<4\pi} |f_k| \left|\Psi_k\left(x\right)\right| + \sum_{\mu_k\geq 4\pi} |f_k| \left|\Psi_k\left(x\right)\right|.$$

By the condition of "sum of units" (see [7])

$$\sum_{\tau\leq\mu_k\leq\tau+1} 1 \leq const, \ \forall\tau\geq 0 \tag{18}$$

and uniform boundedness of the system $\{\Psi_k\left(x\right)\}_{k=1}^{\infty}$ it holds

$$\sum_{0\leq\mu_k<4\pi} |f_k| \left|\Psi_k\left(x\right)\right|$$

$$\leq const \sum_{0\leq\mu_k<4\pi} |f_k| \leq const \left\|f\right\|_{1,m} \sum_{0\leq\mu_k<4\pi} 1 \leq const \left\|f\right\|_{1,m}.$$

For estimating $\sum_{\mu_k\geq 4\pi} |f_k| \left|\Psi_k\left(x\right)\right|$ we apply the lemma, the condition of the "sum of units" (18) and condition (2). As a result, we get

$$\sum_{\mu_k\geq 4\pi} |f_k| \left|\Psi_k\left(x\right)\right| \leq \sum_{\mu_k\geq 4\pi} |f_k| \left\|\Psi_k\right\|_{\infty,m} \leq const \sum_{\mu_k\geq 4\pi} |f_k|$$

$$\leq const \left\{C\left(f\right) \sum_{\mu_k\geq 4\pi} \mu_k^{\alpha-3} + \sum_{\mu_k\geq 4\pi} \mu_k^{-1}\omega_{1,m}\left(f', \mu_k^{-1}\right) + \left\|f'\right\|_{1,m} \sum_{\mu_k\geq 4\pi} \mu_k^{-2}\right.$$

$$\left. + \left(\left\|f\right\|_{\infty,m} + \left\|f'\right\|_{1,m}\right) \sum_{r=2}^{3} \left\|\left\|U_r\right\|\right\|_1 \left(\sum_{\mu_k\geq 4\pi} \mu_k^{-r}\right)\right\}$$

$$\leq const \left\{C\left(f\right) \sum_{n=[4\pi]}^{\infty} n^{-\alpha-3} \left(\sum_{n\leq\mu_k\leq n+1} 1\right)\right.$$

$$+\sum_{n=[4\pi]}^{\infty} n^{-1}\omega_{1,m}\left(f',n^{-1}\right)\left(\sum_{n\le\mu_k\le n+1}1\right)+\left\|f'\right\|_{1,m}\sum_{n=[4\pi]}^{\infty}n^{-2}\left(\sum_{n\le\mu_k\le n+1}1\right)$$

$$+\left(\|f\|_{\infty,m}+\left\|f'\right\|_{1,m}\right)\sum_{r=2}^{3}\|\|U_r\|\|_1\sum_{n=[4\pi]}^{\infty}n^{-r}\left(\sum_{n\le\mu_k\le n+1}1\right)\Bigg\}$$

$$\le const\Bigg\{C(f)[4\pi]^{\alpha-2}+\sum_{n=[4\pi]}^{\infty}n^{-1}\omega_{1,m}\left(f',n^{-1}\right)+\left\|f'\right\|_{1,m}[4\pi]^{-1}$$

$$+\left(\|f\|_{\infty,m}+\left\|f'\right\|_{1,m}\right)\sum_{r=2}^{3}\|\|U_r\|\|_1[4\pi]^{1-r}\Bigg\}<\infty.$$

Thus, the expansion $\sum_{k=1}^{\infty} f_k\Psi_k(x)$ converges absolutely and uniformly on $\overline{G}$. By the completeness of the system $\{\Psi_k(x)\}_{k=1}^{\infty}$, in $L_2^m(G)$ the given expansion converges uniformly namely to the function $f(x)$. Consequently, it holds the equality

$$f(x)=\sum_{k=1}^{\infty}f_k\Psi_k(x),\quad x\in\overline{G}. \tag{19}$$

Estimate the difference $f(x)-\sigma_\nu(x,f)$. For that we use equality (19), uniform boundedness of the system $\{\Psi_k(x)\}_{k=1}^{\infty}$, conditions (2), (18) and the lemma. As a result we get

$$\sup_{x\in\overline{G}}|\sigma_\nu(x,f)-f(x)|=\sup_{x\in\overline{G}}\left|\sigma_\nu(x,f)-\sum_{k=1}^{\infty}f_k\Psi_k(x)\right|$$

$$=\sup_{x\in\overline{G}}\left|\sum_{\mu_k>\nu}f_k\Psi_k(x)\right|\le\sum_{\mu_k\ge\nu}|f_k|\,\|\Psi_k\|_{\infty,m}\le const\sum_{\mu_k\ge\nu}|f_k|$$

$$\le const\sum_{n=[\nu]}^{\infty}\left(\sum_{n\le\mu_k\le n+1}|f_k|\right)\le const\sum_{n=[\nu]}^{\infty}\Bigg(\sum_{n\le\mu_k\le n+1}\Big\{C(f)\mu_k^{\alpha-3}$$

$$+\mu_k^{-1}\omega_{1,m}\left(f',\mu_k^{-1}\right)+\mu_k^{-2}\left\|f'\right\|_{1,m}+\left(\|f\|_{\infty,m}+\left\|f'\right\|_{1,m}\right)\sum_{r=2}^{3}\|\|U_r\|\|_1\mu_k^{-r}\Big\}\Bigg)$$

$$\le const\Bigg\{C(f)\nu^{\alpha-2}+\sum_{n=[\vartheta]}^{\infty}n^{-1}\omega_{1,m}\left(f',n^{-1}\right)+\nu^{-1}\left\|f'\right\|_{1,m}$$

$$+\left(\|f\|_{\infty,m}+\left\|f'\right\|_{1,m}\right)\sum_{r=2}^{3}\|\|U_r\|\|_1\nu^{1-r}\Bigg\}.$$

Consequently, estimation (3) is established. The theorem is proved.

Corollary 1. *If the system $\{\Psi_k(x)\}_{k=1}^{\infty}$ is uniformly bounded, $f(x)\in W_{1,m}^1(G)$, $f(0)=f(1)=0$ and $f'(x)\in H_{1,m}^{\alpha}(G)$, $0<\alpha<1$ ($H_{1,m}^{\alpha}(G)$ is the Nicolsky class), then*

$$\sup_{x\in\overline{G}}|\sigma_\nu(x,f)-f(x)|\le const\,\nu^{-\alpha}\left\|f'\right\|_{1,m}^{\alpha}$$

where

$$\|g\|_{1,m}^{\alpha}=\|g\|_{1,m}+\sup_{\delta>0}\delta^{-\alpha}\omega_{1,m}(g,\delta).$$

Corollary 2. *If the system* $\{\Psi_k(x)\}$ *is uniformly bounded,* $f(x) \in W^1_{1,m}(G)$*,* $f(0) = f(1) = 0$ *and for some* $\beta > 0$ *it is fulfilled the estimation*

$$\omega_{1,m}(f,\delta) = O\left(\ln^{-(1+\beta)} \delta^{-1}\right), \ \ \delta \to +0,$$

then

$$\sup_{x \in \overline{G}} |\sigma_\nu(x,f) - f(x)| = O\left(\ln^{-\beta} \nu\right), \ \nu \to \infty.$$

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