Transactions of National Academy of Sciences of Azerbaijan
Series of Physical-Technical and Mathematical Sciences, vol. XXXV, No 1, 2015.

The Influence Of The Initial Stressed On The Dynamics Of The Moving Load Acting On The Hydro-Elastic System Consisting Of The Pre-Stressed Plate, Compressible Viscous Fluid And Rigid Wall

Maftun I. Ismailov · Kamala A. Salmanova

Received: 15.09.2014 / Revised: 10.12.2014

Abstract. *The paper studies the influence of the initial stress in the plate on the dynamics of the moving load acting on the hydroelastic system consisting of the mentioned plate, compressible viscous fluid and rigid wall. The motion of the plate is described by three-dimensional linearized theory of elastic waves in initially stressed bodies, and the motion of the compressible viscous fluid is described by the linearized Navier-Stokes equations. Numerical results related to the aforementioned influence of the initial stresses are presented and discussed.*

1. Introduction

One of the considerable aspects of the investigations regarding the plate-fluid systems is a dynamic response analysis of these systems induced by a moving load. Results of these investigations are applied for construction of the floating bridges and for determination of their efficiency. As an example for such investigations it can be presented studies carried out in papers [1 - 3] and others listed therein. However in these investigations the fluid reaction to the plate (i.e. to the floating bridge) is taken into consideration without solution of the equations of the fluid motion. Namely, in these works the so-called hydrostatic force (R) caused by the plate-fluid interaction is determined through linear spring model, i.e. through the relation $R = -kw$, where w is a vertical displacement of the plate (i.e. the floating bridge) and k is a spring constant. Consequently, in the foregoing investigations the existence of the fluid is taken into consideration only through this spring constant. It is evident that the approach employed in [1 - 3] is very approximate one and cannot answer the questions how the fluid viscosity, fluid depth, fluid compressibility, plate thickness and the moving velocity of the external force act on the mentioned "hydrostatic force" and fluid flow velocities. To find the answers to these questions it is necessary to solve the corresponding coupled fluid-plate interaction problems within the scope of the exact linearized equations described to the plate and fluid motions. In the mentioned sense, in the present paper the attempt is made for solution to the problems related to the dynamics of the moving load acting on a system consisting of the pre-stressed metal elastic plate, compressible viscous fluid and a rigid wall and numerical results related to the influence of the initial stress in the plate on this dynamic are presented and discussed. It should noted that the forced vibration of the subject under consideration was studied in works [4 – 6]. Moreover, the dynamics of the oscillating moving load acting on the pre-strained highly elastic bi-layered elastic plate resting on the rigid foundation was investigated in a paper [7].

M.I. Ismailov
Nakhchivan State University,
University campus, AZ 7000, Nakhchivan, Azerbaijan
E-mail: imeftun@yahoo.com

K.A. Salmanova
Ganja State University
187, Sh. I. Khatai str., AZ 2000, Ganja, Azerbaijan
E-mail: ksalmanova@yahoo.com

2. Formulation of the problem

Consider a system consisting of the pre-stressed plate-layer, barotropic compressible Newtonian viscous fluid and rigid wall (Fig.1). We associate the coordinate system $Ox_1x_2x_3$ with the plate and the position of the points of the constituents we determine through the Lagrange coordinates in this coordinate system. Below, the values related to the initial state are denoted by the upper index 0. We assume that the initial stress state in the plate is determined within the scope of the classical linear theory of elasticity and this stress state is expressed by the expressions $\sigma_{11}^0 = const$ and $\sigma_{ij}^0 = 0 \ \ if \neq 11$. Within this, we consider a motion of the plate-layer in the case where the lineal-located force which moves with the constant velocity V acts on its free face plane. This consideration will be made within the framework of the three-dimensional linearized theory of elastic waves in initially stressed bodies (TDLTEWISB). Assume that the plate occupies the region $\{|x_1| < \infty, -h < x_2 < 0\}$, but the fluid occupies the region $\{|x_1| < \infty, -h_d < x_2 < -h\}$.

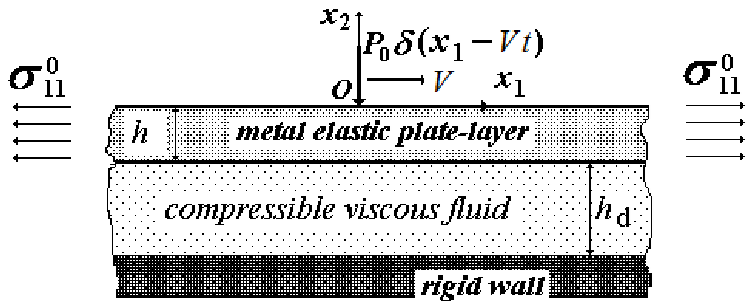

Fig.1.

We use the Murnaghan potential for describing the elasticity relations of the plate material and within the scope of the foregoing assumptions we obtain following field equations of the TDLTEWISB [8] for the compressible body under the plane-strain state in the $Ox_1x_2x_3$ plane.

$$\frac{\partial \sigma_{11}}{\partial x_1} + \frac{\partial \sigma_{11}}{\partial x_1} + \sigma_{11}^0 \frac{\partial^2 u_1}{\partial x_1^2} = \rho \frac{\partial^2 u_1}{\partial t^2}, \frac{\partial \sigma_{12}}{\partial x_1} + \frac{\partial \sigma_{22}}{\partial x2} + \sigma_{11}^0 \frac{\partial^2 u_2}{\partial x_1^2} = \rho \frac{\partial^2 u_2}{\partial t^2},$$

$$\sigma_{11} = \omega_{1111}\varepsilon_{11} + \omega_{1122}\varepsilon_{22}, \ \ \sigma_{22} = \omega_{2211}\varepsilon_{11} + \omega_{2222}\varepsilon_{22}, \ \sigma_{12} = 2\omega_{1212}\varepsilon_{12},$$

$$\omega_{1111} = \lambda + 2\mu + \frac{\sigma_{11}^0}{\mu}(2b + c) + \frac{2\sigma_{11}^0}{3K_0}\left[(a + b) - (2b + c)\frac{\lambda}{2\mu}\right],$$

$$\omega_{2222} = \lambda + 2\mu + \frac{2\sigma_{11}^0}{3K_0}\left[(a + b) - (2b + c)\frac{\lambda}{2\mu}\right],$$

$$\omega_{1122} = \omega_{2211} = \lambda + \frac{b}{\mu}\sigma_{11}^0 + \frac{2\sigma_{11}^0}{3K_0}\left[a - b - \frac{\lambda}{\mu}\right],$$

$$\omega_{1212} = \mu + \frac{b}{3K_0}\sigma_{11}^0 + \frac{c\sigma_{11}^0}{4\mu}\left[\frac{\lambda + 2\mu}{3K_0}\right],$$

$$K_0 = \lambda + \frac{2\mu}{3}, \ \ \varepsilon_{11} = \frac{\partial u_1}{\partial x_1}, \varepsilon_{22} = \frac{\partial u_2}{\partial x_2}, \varepsilon_{12} = \frac{1}{2}\left(\frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1}\right). \qquad (1)$$

Note that in Eq. (1) λ and μ are Lame constants, a, b and c are third order elastic constants of the plate material. Other notation used in (1) is conventional.

According to [9], we consider the field equations of motion of the Newtonian compressible viscous fluid: the density, viscosity constants and pressure of which are denoted by the upper index (1). Under this consideration, according to the smallness of the perturbation of the displacements, we identify the Lagrange coordinates with the corresponding Euler coordinates.

Thus, the linearized Navier-Stokes and other field equations for the fluid are:

$$\rho_0^{(1)}\frac{\partial \nu_i}{\partial t}-\mu^{(1)}\frac{\partial \nu_i}{\partial x_j \partial x_j}+\frac{\partial \rho^{(1)}}{\partial x_i}-\left(\lambda^{(1)}+\mu^{(1)}\right)\frac{\partial^2 \nu_j}{\partial x_j \partial x_i}=0,$$

$$\frac{\partial \rho^{(1)}}{\partial t}+\rho_0^{(1)}\frac{\partial \nu_j}{\partial x_j}=0, T_{ij}=\left(-p^{(1)}+\lambda^{(1)}\theta\right)\delta_{ij}+2\mu^{(1)}e_{ij},$$

$$\theta=\frac{\partial \nu_1}{\partial x_1}+\frac{\partial \nu_2}{\partial x_2},\ e_{ij}=\frac{1}{2}\left(\frac{\partial \nu_i}{\partial x_j}+\frac{\partial \nu_j}{\partial x_i}\right)\cdot a_0^2=\frac{\partial p^{(1)}}{\partial \rho^{(1)}} \tag{2}$$

where $\rho_0^{(1)}$ is the fluid density before perturbation. The other notation used in Eq. (2) is also conventional.

According to [9], the solution of the system equations in Eq. (2) for the 2D plane problems is reduced to finding the two potentials φ and ψ which are determined from the following equations:

$$\left[\left(1+\frac{\lambda^{(1)}+2\mu^{(1)}}{a_0^2\rho_0^{(1)}}\frac{\partial}{\partial t}\right)\Delta-\frac{1}{a_0^2}\frac{\partial^2}{\partial t^2}\right]\varphi=0,$$

$$\left(\nu^{(1)}\Delta-\frac{\partial}{\partial t}\right)\psi=0,\ \ \Delta=\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_2^2}, \tag{3}$$

where $\nu^{(1)}$ is the kinematic viscosity, i.e. $\nu^{(1)}=\mu^{(1)}/\rho_0^{(1)}$.

The velocities ν_1, ν_2 and the pressure $p^{(1)}$ are expressed by the potentials φ and ψ through the expressions

$$\nu_1=\frac{\partial \varphi}{\partial x_1}+\frac{\partial \psi}{\partial x_2},\ \ \nu_2=\frac{\partial \varphi}{\partial x_2}-\frac{\partial \psi}{\partial x_1},$$

$$p^{(1)}=\rho_0^{(1)}\left(\frac{\lambda^{(1)}+2\mu^{(1)}}{\rho_0^{(1)}}\Delta-\frac{\partial}{\partial t}\right)\varphi. \tag{4}$$

Assuming that

$$p^{(1)}=-\left(T_{11}+T_{22}+T_{33}\right)/3, \tag{5}$$

we obtain:

$$\lambda^{(1)}=-\frac{2}{3}\mu^{(1)}. \tag{6}$$

Moreover, it is assumed that the following boundary, contact and impermeability conditions are satisfied:

$$\sigma_{21}|_{x_2}=0,\ \sigma_{22}|_{x_2}=0=-P_0\delta\left(x_1-Vt\right),\ \left.\frac{\partial u_1}{\partial t}\right|_{x_2=-h}=\left.\nu_1\right|_{x_2=-h},$$

$$\left.\frac{\partial u_2}{\partial t}\right|_{x_2=-h}=\left.\nu_2\right|_{x_2=-h},$$

$$\left.\sigma_{21}\right|_{x_2=-h}=\left.T_{21}\right|_{x_2=-h},\ \left.\sigma_{22}\right|_{x_2=-h}=\left.T_{22}\right|_{x_2=-h},\ \left.\nu_1\right|_{x_2=-h-h_d}=0,$$

$$\left.\nu_2\right|_{x_2=-h-h_d}=0, \tag{7}$$

where $\delta\left(\,\right)$ is the Dirac delta function.

This completes the formulation of the problem.

3. Method of solution

For the solution of this problem, we use the moving coordinate system $x_1'=x_1-Vt, x_2'=x_2$ (below we will omit the upper prime on the new moving coordinates) and replacing the derivatives $\partial\left(\cdot\right)/\partial t$ and $\partial^2\left(\cdot\right)/\partial t^2$ with $-V\frac{\partial}{\partial x_1}$ and $V^2\frac{\partial^2}{\partial x_1^2}$, respectively, we obtain the corresponding equations and boundary and contact conditions for the sought values in the moving coordinate system. For the solution to these equations, we employ the exponential Fourier transformation with respect to the x_1 coordinate

$$f_F(s,x_2)=\int_{-\infty}^{+\infty} f(x_1,x_2)\,e^{-isx_1}dx_1 \tag{8}$$

to these equations. The inverse of the Fourier transformation of the sought values can be represented as follows:

$$\frac{1}{2\pi}\int_{-\infty}^{+\infty}\{u_{1F};u_{2F};\sigma_{11F};\sigma_{12F};\sigma_{22F};\nu_{1F};\nu_{2F};T_{11F};T_{12F};T_{22F}\}\,e^{isx_1}ds. \tag{9}$$

Before the employing the Fourier transformation (8) we introduce the dimensionless coordinates and dimensionless transformation parameter

$$\overline{x}_1=x_1/h,\overline{x}_2=x_2/h,\overline{s}=sh. \tag{10}$$

Below we will omit the over-bar on the symbols in (10). Moreover, we will also use the notation

$$V'=V/h,\quad \nu^{(1)}=\mu^{(1)}/\rho_0^{(1)}. \tag{11}$$

First, we consider the solution of the equations related to the Fourier transformation of the quantities related to the plate-layer, i.e. to the solution of the equations which are obtained from the equations (1) and (9). Thus, substituting the expressions (9) into the equations (1), and doing some mathematical manipulations we obtain the following equations for u_{1F} and u_{2F}.

$$Au_{1F}-B\frac{du_2F}{dx_2}+C\frac{d^2u_1F}{dx_2^2}=0,\ \ Du_{2F}+B\frac{du_1F}{dx_2}+G\frac{d^2u_2F}{dx_2^2}=0 \tag{12}$$

where

$$A=X^2-s^2\omega_{1111},\ \ B=s\left(\omega_{1122}+\omega_{2121}\right),\ \ C=\omega_{2112},$$
$$D=s^2\left(X^2-\omega_{1221}\right),\ \ G=\omega_{2222},\ X^2=V'^2h^2/c_2^2,\ c_2=\sqrt{\mu/\rho}. \tag{13}$$

Introducing the notation

$$A_0=\frac{AG+B^2+CD}{CG},\ B_0=\frac{BD}{CG},$$
$$k_1=\sqrt{-\frac{A_0}{2}+\sqrt{\frac{A_0^2}{4}-B_0}},\ k_2=\sqrt{-\frac{A_0}{2}-\sqrt{\frac{A_0^2}{4}-B_0}}, \tag{14}$$

we can write the solution to the equation (12) as follows:

$$u_{2F}=Z_1e^{k_1x_2}+Z_2e^{-k_1x_2}+Z_3e^{k_2x_2}+Z_4e^{-k_2x_2},$$
$$u_{1F}=Z_1a_1e^{k_1x_2}+Z_2a_2e^{-k_1x_2}+Z_3a_3e^{k_2x_2}+Z_4a_4e^{-k_2x_2}, \tag{15}$$

where

$$a_1=\frac{-D-Gk_1^2}{Bk_1^2},\ \ a_2=-a_1,\ \ a_3=\frac{-D-Gk_2^2}{Bk_2^2},\ a_4=-a_3. \tag{16}$$

Now we consider determination of the Fourier transformations of the quantities related to the fluid flow. First, we consider the determination of φ_F and ψ_F from the Fourier transformation of the equations in (3). Taking the relation

$$\varphi_F=-sV'h^2\widetilde{\varphi}_F,\ \ \psi_F=-sV'h^2\widetilde{\psi}_F \tag{17}$$

into account, it can be written that

$$\frac{d^2\widetilde{\varphi}_F}{dx_2^2}+s^2\left(\frac{\Omega_1^2}{1-i4s\Omega_1^2/\left(3N_w^2\right)}-1\right)\widetilde{\varphi}_F=0,\frac{d^2\widetilde{\psi}_F}{dx_2^2}+\left(s^2-isN_w^2\right)\widetilde{\psi}_F=0, \tag{18}$$

where

$$\Omega_1 = \frac{V'h}{a_0}, \quad N_w^2 = \frac{V'h^2}{v^{(1)}}. \tag{19}$$

The dimensionless number N_w in (19) can be taken as a Womersley number and characterizes the influence of the fluid viscosity on the mechanical behavior of the system under consideration. The dimensionless frequency Ω_1 in (19) can be taken as the parameter through which the influence of the compressibility of the fluid on the mechanical behavior of the system under consideration can be characterized.

Thus, taking the crelation (6) into consideration, the solutions to the equations in (18) are found as follows:

$$\widetilde{\varphi}_F = Z_5 e^{\delta_1 x_2} + Z_7 e^{-\delta_1 x_2}, \quad \widetilde{\psi}_F = Z_6 e^{\gamma_1 x_2} + Z_8 e^{-\gamma_1 x_2}, \tag{20}$$

where

$$\delta_1 = s^2 \sqrt{1 - \frac{\Omega_1^2}{1 - i4s\Omega_1^2 / \left(3N_w^2\right)}}, \quad \gamma_1 = \sqrt{s^2 - isN_w^2}. \tag{21}$$

Using (21) and (18) we obtain the following expressions for the velocities, pressure and stresses of the fluid from the Fourier transformations of the Eqs. (2) and (3)

$$\nu_{1F} = -sV'h\left[-Z_5 s e^{\delta_1 x_2} - Z_7 s e^{-\delta_1 x_2} + Z_6 e^{\gamma_1 x_2} + Z_8 e^{-\gamma_1 x_2}\right],$$

$$\nu_{2F} = -sV'h\left[Z_5 \delta_1 e^{\delta_1 x_2} - Z_7 \delta_1 e^{-\delta_1 x_2} - Z_6 s e^{\gamma_1 x_2} - Z_8 e^{-\gamma_1 x_2}\right]. \tag{22}$$

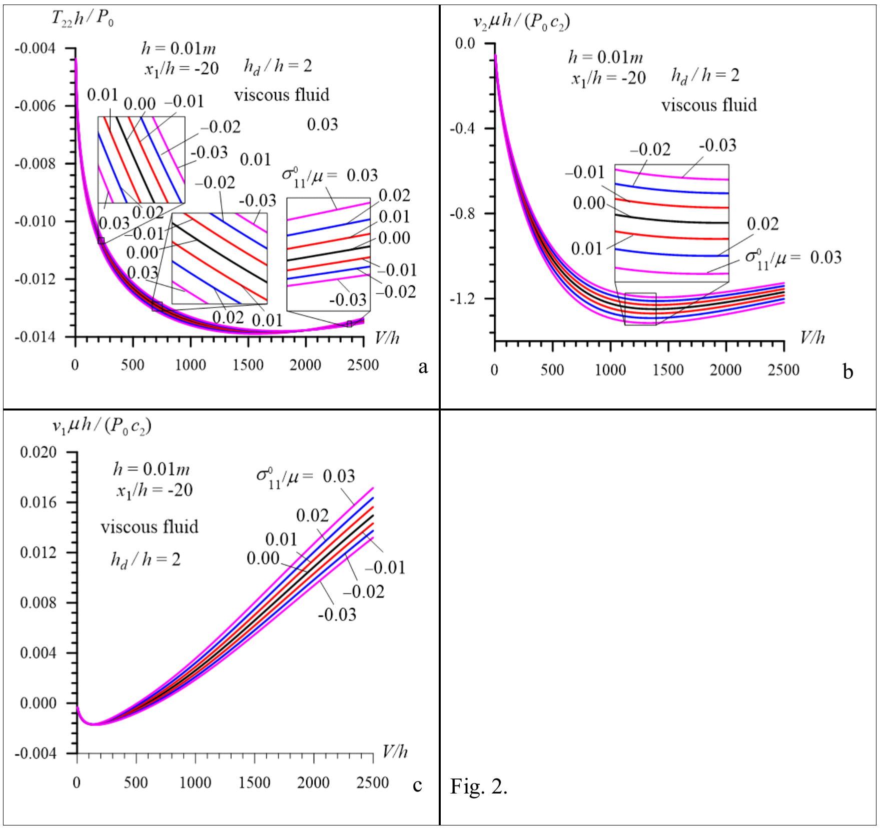

Fig. 2.

Using the solutions (15) and (22), and the expressions given in (1) and (2), we obtain a system of equations with respect to the unknowns $Z_1, Z_2, \ldots, Z_8$ from the boundary and contact conditions in (7). Thus, after determination of these unknowns we can determine the sought values through the following relation:

$$\{u_1; u_2; \sigma_{11}; \sigma_{22}; \nu_1; \nu_2; T_{11}; T_{12}; T_{22}\}$$

$$= \frac{1}{2\pi} \mathrm{Re} \left[\int_{-\infty}^{+\infty} \{u_{1F}; u_{2F}; \sigma_{11F}; \sigma_{12F}; \sigma_{22F}; \nu_{1F}; \nu_{2F}; T_{11F}; T_{12F}; T_{22F}\} \, e^{isx_1} ds \right]. \quad (23)$$

The algorithm employed for calculation of the integrals in the form (23) was discussed in the papers [3 -6] and therefore here we do not consider again this discussion

4. Numerical results and discussions

In the numerical investigation we assume that the material of the plate-layer is Steel with mechanical constants: $\mu = 79 \times 10^9 Pa$, $\lambda = 94.4 \times 10^9 Pa$, $a = -235 \times 10^9 Pa$. $b = -275 \times 10^9 Pa$, $c = -490 \times 10^9 Pa$ and density $\rho = 1160 kg/m^3$ [8], but the material of the fluid is Glycerin with viscosity coefficient $\mu^{(1)} = 1,393 kg/(m \cdot s)$, density $\rho = 1260 kg/m^3$ and sound speed $a_0 = 1927 m/s$ [9]. We also introduce the notation $c_2 = \sqrt{\mu/\rho}$ which is the shear wave propagation velocity in the layer material in the case where the initial strains are absent.

Thus, after selection of these materials, the foregoing dimensionless parameters can be determined through the three quantities: h (the thickness of the plate-layer), h_d (the thickness of the fluid strip), V (the velocity of the external moving load) and σ_{11}^0 (the initial stress in the layer). Numerical results which will be discussed below relate to the normal stress acting on the interface plane between the fluid and plate-layer and to the velocities of the fluid (or of the plate-layer) on the mentioned interface plane in the directions of the Ox_1 and O_{x_2} axes (Fig. 1).

Thus we consider the numerical results which illustrate how the initial stretching or compressing of the plate along the Ox_1 axis acts on the distribution of the studied stress and velocities. These results are given in Fig. 2 which show the dependence among $T_{22}h/P_0$ (Fig. 2a), $\nu_2 \mu h/(P_0 c_2)$ (Fig. 2b), $\nu_1 \mu h/(P_0 c_2)$ (Fig. 2c) and V/h for the viscous fluid case in the various values of the σ_{11}^0/μ under $h = 0,01m$, $h_d/h = 2$ and $x_1/h = -20.0$. According to these results, we can conclude that the initial stretching or compressing of the plate causes to change the stress and velocities quantitative sense only. For instance, in the case under consideration the initial stretching (compressing) of the plate caused to increase (to decrease) the absolute values of the velocities $\nu_2 \mu h/(P_0 c_2)$ and $\nu_1 \mu h/(P_0 c_2)$.

References

1. Wu J.S., Shih P.Y.: *Moving-load-induced vibrations of a moored floating bridge*. Computer & Structures, **66**, No 4, 435 – 461 (1998).
2. Fu S., Cui W.,Chen X., Wang C.: *Hydroelastic analysis of a nonlinearity connected floating bridge subjected to moving loads.* Marine Structures, **18**, 85 – 107 (2005).
3. Wang C., Fu S., Cui W.: *Hydroelasticity based fatigue assessment of the connector for a ribbon bridge subjected to a moving load.* Marine Structures, **22**, 246 – 260 (2009).
4. Akbarov S.D., Ismailov M.I.: *Forced vibration of a system consisting of a pre-strained highly elastic plate under compressible viscous fluid loading.* CMES: Computer Modeling in Engineering & Science, **97** No 4, 359 – 390 (2014).
5. Akbarov S.D., Ismailov M.I.: *Frequency response of a viscoelastic plate under compressible viscous fluid loading.* International Journal of Mechanics, **8**, 332 – 344 (2014).
6. Akbarov S.D., Ismailov M.I.: *On the forced oscillations of a hydroelastic system consisting of a metallic plate with initial stresses, compressible viscous fluid and absolute rigid wall.* Reports of National Academy of Sciences of Azerbaijan, **LXX**, No 3, 24 – 28 (2014).
7. Akbarov S.D., Salmanova K.A.: *On the dynamics of a finite pre-strained slab resting on a rigid foundation under action of an oscillatinf moving load.* J. Sound and Vibr, **327**, No (3/5), 454 - 472 (2009).
8. Guz A.N.: *Elastic waves in bodies with initial (residual) stresses.* A.C.K., Kiev (2004), in Russian.
9. Guz A.N.: Dynamics of compressible viscous fluid. Cambridge Scientific Publishers: (2009).