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Analysis Of A Finite Span Rectilinear Winglets Influnce On Its Induced Drag By The Distributed Vortex Method

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Abstract. *The article presents the mathematical model of the winglets influence on the induced drag with the eddy method. Considering the output of prefabricated eddies from winglets, P-shaped schemes of eddies are formed on the top of the wing at the level of winglets. As in conventional wings the distance between the center of free eddies is taken slightly more than wingspan of the winglets. Using the Bio-Savar formula inductive velocity distribution created by free eddies at the level of wings is written and the average inductive speed of wingspan is found. On the basis of this the angle of refraction of the flow and induced force of drag is determined. Indentified formulas indicate the decrease of induced drag.*

1. Introduction. A wing creating a lift forms air rarefaction area on its upper surface and compression area under lower one that causes pressure drop and overflow of air at the edge of the wing. However, the overflow through the leading and trailing edge of a wing is impossible since near the edges, pressure over and under the wing is equalized. Near the edges of a wing such a overflow is realized, air from under the wing overflows to its upper surface. Under the wing the motion is directed to the edges of the wing, while under the wing to its middle part. Speed component directed perpendicularly to the moving flow along the wing is formed. These currents bend the stream line of the basic flow going in front. As a result of interactions of these lines, it is formed a vortex wake being a totality of eddy lines originating in the trailing edge of the wing. The vortex lines constituting the eddy wake are called free eddies. Interaction of upper and lower flows near the covering edge of the wing causes rotation of air particles that are taken away with moving flow and form two so called free eddies of great intensity having opposite rotations. Existence of the wingtips moves aside these free eddies from the edges of the wing and so decreases the created by it vertical speeds (induced speed) of moving flow under the wing. The length of winglets and cruising speed don't allow lateral flows to meet under and over the wing. Free eddies originating from the pavement of edges almost parallelly move off back and create vortical trace. There exists the notion of "eddy cord" behind the wing and wake. The wakes are formed at continuous flow about body in viscous medium and are stipulated by drag resistance of the body. Eddy cords are formed at flow of a body in medium because of origination of lift. Intensity of eddies depends on the state of atmosphere, aerodynamic arrangement and airplane configuration, flight mass, speed and height of flight. The eddies form a field of perturbed speeds characterized by the components: V_x are axial speeds, V_R radial speeds, V_t are peripheral (tangential) speeds expressed by vertical V_y and horizontal V_z speeds, i.e. $V_t = \sqrt{V_y^2 + V_z^2}$. The thickness of eddy cors are commensurable with the thickness of turbulent boundary layer coming down from the sharp trailing pavement edge of the wing. Under the radius of eddy core one understands the distance from the axis of the eddy core to the point in its lateral cross-section, where the value of peripheral speed is maximum.

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Joukowsky using the model of ideal fluid, suggested to look for the source of force action of the flow on the body in formation of circulation. Formation of circulation may be represented as the result of influence of eddy arranged in the center of the washing body on flow. According to Joukskii's theorem the lift of infinite span wing is proportional to circulation of speed along the contour enveloping the wing. Therefore, the wing may be changed be infinitely long eddy cord by the circulation of speed of the same size as of the wing. Such an eddy cord is called adjoined. Closing a pair of free eddies that begin at the end of span and under right angle goes back from the wing, by the part of the adjoined eddy on the wing, one gets Π-shaped eddy scheme equivalent to finite span wing [1-15]. Such a vortex system agrees with the Helmholtz theorem according to which the adjoined eddy may not be finished at its tips. By the same Helmholtz theorem, at this scheme of vortices the circulation Γ around the wing at all its cross-sections and around each of free eddies will be one and the same. This eddy system induces in the air flow additional speeds and causes down-wash.

Note that wake vortices of an aircraft were investigated in a number of papers as [15-29].

These investigations is of great interest for us in order to determine the inductive speeds under the wing and in continuation of the wing. In the monograph [15], numerous results of theoretical and experimental investigations are given in the form of graphs. Vortex wake turbulence based on Navier – Stocke's equations with direct numerical modeling were studied in the papers [16-19]. Simulation of large vortices with using Navier-Stocke's equations and subgrid turbulence model were investigated in [20-25].

The papers [26-29] were devoted to numerical solution of Reynolds averaged Navier-Stocks equations jointly with one of differential turbulence models.

By the Joukowsky theorem, lifting force of the area of single span infinite wing equals $Y = \rho_\infty V_\infty \Gamma$, where ρ_∞ and V_∞ are density and speed of undisturbed airflow, Γ is circulation of speed. For a finite span wing it is proved that in the existence of circulation along the contour L covering the wing, the vortices go away from the wing. Indeed, on the contour L construct some surface S not interesting the wing. Circulation of speed Γ on the closed contour L is determined by the formula $\Gamma = \oint\limits_L \mathbf{V} d\mathbf{l}$. By the Stock theorem, it equals the intensity of the vortex $\Gamma = 2\iint\limits_S \omega_n dS$, where ω_n is the projection of angular speed in the direction of the normal n to the area dS. Because of variability of moving flow speed around the wing creating lifting force, the speed circulation differs from zero $\Gamma \neq 0$. Therefore, on any surface S constructed in the above mentioned way, there should exist the points at which $\omega_n \neq 0$. In other words, if circulation around the wing differs from zero, i.e. if the wing creates a lifting force, outside the wing there should exist the vortices penetrating the surface S. This situation, with respect to the existence of vortices outside the wing allows to construct flow models that approximately reflect the pattern of flow around the wing.

The inductive speed V_y is directed along the normal to the vector of speed of moving unperturbed flow V_∞. This movement is superimposed on front oncoming sustained flow and it leads to the fact that flow ahead of the wing and also underneath deflects downwards at an angle $\Delta\alpha = \arg tg V_y/V_\infty$ called the bevel angle. Note that the bevel angle $\Delta\alpha$ and speed V_y in the general case have variable values both along the wingspan and along its chord. Accordingly, the true aerodynamic angle of attack of the considered cross sections $\alpha_{ist} = \alpha_\infty - \Delta\alpha$ diminishes. Rotation down of speed vector of unperturbed flow on the bevel angle causes the same back rotation of the lifting force as it is always perpendicular to the true speed vector. The component of lifting force to the side opposite to the wing movement is called induced (eddy) drag. It is not associated with the property of viscosity of the medium. The induced drag as a lifting force generating it, is created by pressure drop under and on the wing and vanishes at zero lifting force. Physical occurrence of induced drag is stipulated by losses of a part of kinetic energy of the moving wing, spended in the formation of vortices trailing from its edges.

In the case of subsonic flight regime at unseparated flow, the complete drag resistance factor of the wing is represented in the form of the sum of the factors of profile and induced drags: $C_x = C_{xp} + C_{xi}$. The induced drag factor is expressed by the formula

$$C_{xi} = \frac{C_y^2}{\pi\lambda}(1+\delta) \tag{1}$$

where δ is a small value dependent on the wing shape in the plane. For a rectilinear wing $\delta \approx 0,05$. For other wings this value is less. As is seen, its elongation $\lambda \to \infty$ then $C_{xi} \to 0$, or in other words, the wing turns into a profile, the winglets tips vanish, downwash vanishes, V_y vanishes.

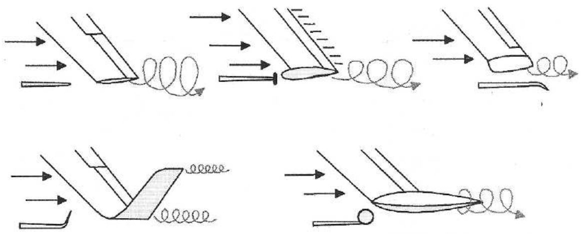

Fig. 1.1 The forms of winglets.

2. Problem Statement

Any drag requires some energy consumption for its overcome or in other words, is associated with loss of kinetic energy of moving flow with transition of mechanical energy to thermal one. Therefore, reduction in drag is an urgent problem of aerodynamics. In practice, it was succeeded to overcome induced drag. In this case in contemporary aircrafts is established the wingtlets (fig. 1) that create aerodynamical force so that the induced drag vector is directed toward the stream. As a result, the total induced drag force of the wing diminishes. The attack angle of the winglet is established so that the maximal efficiency is achieved in the cruising flight of the aircraft. Despite of the fact that the wingtlets are used long ago, we can't see calculations (both analytic and numerical) of their influence on aerodynamical characteristics of the wing neither in textbooks nor in monographs and in scientific papers.

For obtaining a simplest result on efficiency of winglets on aerodynamical characteristics of the wing we assume that the rectilinear winglets in the plane have the length r and chord b equal to the chord of the wing. By the Bio-Savar formula, the speed induced by the vortex cord at a certain point is proportional to the distance of this point from the vortex axis.

Therefore, remove of vortex cords from the winglets diminishes the induced speeds under the wings. The winglets that will be used in this paper at their mathematical simulation have been calculated to this effect.

3. Problem solution.

Let's consider a rectilinear wing of span l and chord b. Denote by r the height of the winglets. We suppose that free vortices arise in the level of the ends of winglets and go with the flow. Denote the distance between the vortices by $l_1 = l + 2e$ where e is some positive number. Depending on the shape of the wing, the ratio e/l changes in the interval $0,01 \div 0,02$. We close these vortices with adjoined vortex of length l_1 located over the wing at height r, parallel to the wing. Locate the origin of coordinates at the centre of the left free vortex and direct the axis Oz along the span to the right, the axis Oy upwards and the axis Ox on the undisturbed flow.

Take any point A with the coordinate z on the adjoined vortex and draw a perpendicular from this point to the wing. The foot of normal on the wing will be the point A' with the coordinates $A'\,(0, -r, z)$. The distance of this point from the centre of the left free vortex will be $OA' = \sqrt{z^2 + r^2}$. As the left free vortex is an infinite halfcord, the induced speed V_y by this vortex at the point A', according to the Bio-Savar formula [12] will be

$$V_y = -\frac{\Gamma}{4\pi\sqrt{z^2 + r^2}}. \tag{3.1}$$

The negative sign shows that the speed V_y is directed downwards, the axis $y-$upwards. For estimating the influence of both vortices, we introduce the induced speed w average in span. The speed average in span from the left free vortex obviously equals

$$\frac{1}{l}\int\limits_{e}^{l+e} V_y dz.$$

By the symmetry, the value of the average speed from the right vortex will be the same, and therefore the average induced speed

$$w = \frac{2}{l}\int\limits_{e}^{l+e} V_y dz. \tag{3.2}$$

Substituting in formula (3.2) the expression of the speed V_y from (3.1), for the speed we have the expression

$$w = \frac{-\Gamma}{2\pi l}\int\limits_{e}^{l+e} \frac{dz}{\sqrt{z^2+r^2}}. \tag{3.3}$$

Using the value of the undetermined integral

$$\int \frac{dx}{\sqrt{x^2+\lambda}} = \ln\left|x+\sqrt{x^2+\lambda}\right| + C$$

after calculating integral (3.3), we get

$$w = \frac{-\Gamma}{2\pi l}\ln\left|\frac{l+e+\sqrt{(l+e)^2+r^2}}{e+\sqrt{e^2+r^2}}\right|. \tag{3.4}$$

So, we obtained the speed w induced by free vortices moving aside form the ends of the winglets. It is easy to prove that at no winglets, i.e. in the case $r=0$, formula (3.4) turns into the formula for induced speed in the case of the wing without winglets that exists in all the books on aerodynamics [1-15].

Prove that the modulus of the function w determined by formula (3.4) is a monotonically decreasing function of the length of the winglet r. To this end calculate the derivative of the function w with respect to r

$$\frac{dw}{dr} = \frac{-\Gamma}{2\pi l}\frac{1}{\left[l+e+\sqrt{(l+e)^2+r^2}\right]\left(e+\sqrt{e^2+r^2}\right)}$$

$$\times\left\{\frac{r}{\sqrt{(l+e)^2+r^2}}\left(e+\sqrt{e^2+r^2}\right) - \frac{r}{\sqrt{e^2+r^2}}\left[l+e+\sqrt{(l+e)^2+r^2}\right]\right\}.$$

After calculation we get

$$\frac{dw}{dr} < -\frac{\Gamma}{2\pi l}\frac{rl}{\sqrt{l^2+r^2}}\frac{1}{\left[l+e+\sqrt{(l+e)^2+r^2}\right]\left(e+\sqrt{e^2+r^2}\right)} < 0.$$

Thus, the velocity w induced by free vortices in modula is a monotonically decreasing function of the height of winglets r and has the greatest value

$$|w| = \frac{\Gamma}{2\pi l}\ln\left|\frac{l+e}{e}\right|$$

for $r=0$, i.e. in the case of no winglets.

Represent the function (3.4) in the form

$$w = \frac{-\Gamma}{2\pi l}\ln\left|\frac{(l+e)\left[1+\sqrt{1+r^2(l+e)^{-2}}\right]}{e\left(1+\sqrt{1+r^2e^{-2}}\right)}\right|$$

$$= \frac{-\Gamma}{2\pi l} \left| \ln \frac{l+e}{e} + \ln \frac{1+\sqrt{1+r^2 (l+e)^{-2}}}{1+\sqrt{1+r^2 e^{-2}}} \right|. \tag{3.5}$$

The right side of this formula consists of the sum of two terms: the first of them corresponds to the induced drag for the wing without winglets, and the second one expresses the influence of winglets. It is easy to see that the fraction under the logarithm of the second term is les than a unit, i.e. this term is a negative value and shows the induced speed reduction. As it was noted above, depending on the winge shape the ratio e/l changes in the interval $0,01 \div 0,02$. And for the first logarithm we get the estimation

$$\ln \left| \frac{l+e}{e} \right| \approx 4\,(1+\delta)\,\frac{l+2e}{l}. \tag{3.6}$$

Assuming in addition, $r = e$, it is easy to get the estimation of the second logarithm

$$\ln \frac{1+\sqrt{1+r^2 (l+e)^{-2}}}{1+\sqrt{1+r^2 e^{-2}}} \approx \ln \frac{2}{1+\sqrt{2}}. \tag{3.7}$$

Assuming $e/l = 0,01$, we easily find that the ratio of values (3.7) and (3.6) are approximately of order 4,5%, and provided $r = 2e$, $e/l = 0,01$ about 7,5 %.

From the equality $tg\Delta\alpha \approx \Delta\alpha = -\frac{w}{V_\infty}$ and formula (3.5) we determine the bevel angle

$$\Delta\alpha = \frac{\Gamma}{2\pi l V_\infty} \left[\ln \frac{l+e}{e} - \ln \frac{1+\sqrt{1+r^2 e^{-2}}}{1+\sqrt{1+r^2 (l+e)^{-2}}} \right]. \tag{3.8}$$

Here in the second logarithm the negative sign is brought forward and the fraction is written in inverse proportion.

Determine the circulation Γ from the condition of equality of the value of lifting force written accordingly by dynamic pressure and circulation

$$Y_a = C_y \frac{\rho V_\infty^2}{2} S = \rho V_\infty \Gamma l. \tag{3.9}$$

Hence we find

$$\Gamma = \frac{1}{2l} C_y V_\infty S.$$

Substituting this expression in formula (3.8), we get the bevel angle

$$\Delta\alpha = \frac{C_y S}{4\pi l^2} \left[\ln \frac{l+e}{e} - \ln \frac{1+\sqrt{1+r^2 e^{-2}}}{1+\sqrt{1+r^2 (l+e)^{-2}}} \right].$$

Taking into account that the ratio l^2/S is the aspect ratio of the wing λ, we rewrite the last formula in the form

$$\Delta\alpha = \frac{C_y}{4\pi\lambda} \left[\ln \frac{l+e}{e} - \ln \frac{1+\sqrt{1+r^2 e^{-2}}}{1+\sqrt{1+r^2 (l+e)^{-2}}} \right].$$

Then the force of induced drag will have the expression

$$X_i = Y_a \Delta\alpha = Y_a \frac{C_y}{4\pi\lambda} \left[\ln \frac{l+e}{e} - \ln \frac{1+\sqrt{1+r^2 e^{-2}}}{1+\sqrt{1+r^2 (l+e)^{-2}}} \right].$$

Using the representation of lifting force and induced drag force through the dynamic pressure

$$Y_a = C_y \frac{\rho V_\infty^2}{2} S, \quad X_i = C_{xi} \frac{\rho V_\infty^2}{2} S,$$

we find the expression of the induced drag factor C_{xi}

$$C_{xi} = \frac{C_y^2}{4\pi\lambda}\left[\ln\frac{l+e}{e} - \ln\frac{1+\sqrt{1+r^2e^{-2}}}{1+\sqrt{1+r^2\left(l+e\right)^{-2}}}\right].$$

Taking into account formula (3.6), we can write

$$C_{xi} = \frac{C_y^2}{\pi\lambda}\left[(1+\delta) - \frac{1}{4}\ln\frac{1+\sqrt{1+r^2e^{-2}}}{1+\sqrt{1+r^2\left(l+e\right)^{-2}}}\right]. \quad (3.10)$$

Here the first term inside the square bracket corresponds to the induced drag factor of a simple wing, and the second one reflects the influence of winglets on the induced drag factor. As is seen, the induced drag factor found in the form of formula (3.10) is less than the same one for a wing without winglets.

Obviously, the ratio of the second term to the first one inside the square bracket in formula (3.10) as above , is estimated approximately about 4,5 % for $r = e, e/l = 0,01$ and about 7,5% for $r = 2e,\ e/l = 0,01$.

We call the modulus of the second term in formula (3.10) the winglet function. This formula allows to make the following conclusions:

- With an increase in lift, bevel angle flow, chord profile and height of winglet, the winglet function increases, and consequently, the induced drag decreases;

- Decrease of C_{xi} is proportional to C_y^2 and this dependence is monotonic;

- With an increase of the wingspan the winglet function decreases as the induced drag itself. Herewith the fraction under the logarithm in formula (2.11) tends to constant value $\frac{1+\sqrt{1+r^2e^{-2}}}{2}$.

- In the model considered in the present paper, the increase of the drong factor at the expense of the existence of winglets at small C_y [30] doesn't happen.

The results obtained in this item, having independent value in aerodynamics of low speed (incompressible flow) of flying vehicles, may be used for aerodynamic research at great speeds (subsonic).

Conclusion. In the paper based in distributed vortices method, a mathematical model on investigation of influence of the winglets on inductive drag of a finite length rectilinear wing is suggested. The obtained formulae show the decrease of inductive drag at the expense of winglets. The average bevel angle of flow under the wing and induced drag are determined. The induced drag is determined as the product of lift by the bevel angle. The formula for the induced drag factor is obtained. It is shown that the induced speed (and also the induced drag force) is a monotonically decreasing function of the height of winglets.

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