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Free oscillations of the longitudinal reinforced rib systems and loaded by with axial compressive forces of the orthotropic cylindrical shell with flowing liquid

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Abstract. In this paper the free oscillations of the longitudinal reinforced rib systems and loaded by axial compressive forcesof the orthotropic cylindrical shell with flowing liquid are investigated. The problem is solved by using Ostroradskii- Hamilton's principle of stationarity of action. The frequency equations of the considred system are realized numerically.

Keywords. Stress, isotropic cylindrical shells, viseoelastic medium, deformation, displacement.

Mathematics Subject Classification (2010): 74J05

Thin-walled shell constructions contacting with a mediumare widely used in rocket, aircraft, shipbuilding, engineering and construction. To give greater rigidity to the thin-walled part of the shell it is reinforced by ribs that significantly increases its strength at a slight increase of the construction mass even if the ribs have small height. Calculations of strength, oscillations and stability of such constructions have an important role at the projecting of modern devices, machines and equipment.

The monograph [3] is on the research on stability and vibrations of reinforced isotropic shells without any medium at static and dynamic loading. Asymptotic analysis of the eigen frequencies of non-axisymmetric and axisymmetric vibrations reinforced by orthotropic cylindrical shells in an infinite elastic medium filled with liquid is carried out in works [4, 9]. Free oscillations of an isotropic cylindrical shell with flowing liquid and in an infinite elastic medium reinforced by longitudinal and cross rib system are studied in works [1, 2]. In works [5, 10] the problem on free oscillations of the reinforced longitudinal and cross rib system and loaded with axial compressive forces of the isotropic cylindrical shells filled with a medium is investigated. Parametrical oscillations of non-linear and non-homogeneous by thickness viscoelastic cylindrical shell and bars contacting with viscoelastic medium are investigated in works [7, 8].

The actual work is devoted to the investigation of free oscillations reinforced by longitudinal ribs system and loaded with axial compressive forces of orthotropic cylindrical shell with flowing liquid.

1 Statement of the problem

The total energy of elastic deformation of the orthotropic cylindrical shells loaded with axial compressive forces has the form:

$$J = \frac{1}{2}R^2 \int_{x_1}^{x_2} \int_{y_1}^{y_2} \left\{N_{11}\varepsilon_{11} + N_{22}\varepsilon_{22} + N_{12}\varepsilon_{12} - M_{11}\chi_{11} - M_{22}\chi_{22} - M_{12}\chi_{12}\right\} dxdy +$$

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$$+\frac{1}{2}\sum_{i=1}^{k_1}\int_{x_1}^{x_2}\left[E_iF_i\left(\frac{\partial u_i}{\partial x}\right)^2+E_iJ_{yi}\left(\frac{\partial^2 w_i}{\partial x^2}\right)^2+E_iJ_{zi}\left(\frac{\partial^2 v_i}{\partial x^2}\right)^2+G_iJ_{kpi}\left(\frac{\partial \varphi_{kpi}}{\partial x}\right)^2\right]dx+$$

$$+\rho_0 h\int_{x_1}^{x_2}\int_{y_1}^{y_2}\left[\left(\frac{\partial u}{\partial t}\right)^2+\left(\frac{\partial v}{\partial t}\right)^2+\left(\frac{\partial w}{\partial t}\right)^2\right]dxdy+ \tag{1.1}$$

$$+\sum_{i=1}^{k_1}\rho_iF_i\int_{x_1}^{x_2}\left[\left(\frac{\partial u_i}{\partial t}\right)^2+\left(\frac{\partial v_i}{\partial t}\right)^2+\left(\frac{\partial w_i}{\partial t}\right)^2+\frac{J_{kpi}}{F_i}\left(\frac{\partial \varphi_{kpi}}{\partial t}\right)^2\right]dx-$$

$$-\int_{x_1}^{x_2}\int_{y_1}^{y_2}\left(q_x u+q_y\vartheta+q_z w\right)dxdy-$$

$$-\frac{\sigma_x h}{2}\int_0^{x_1}\int_0^{2\pi}\left(\frac{\partial w}{\partial x}\right)^2 dxdy-\frac{\sigma_x F_c}{2R}\sum_{i=1}^{k_1}\int_0^{x_1}\left(\frac{\partial w}{\partial x}\right)^2\Bigg|_{y=y_i}dx$$

where $B_{11}=\frac{E_1}{1-\nu_1\nu_2}$;$B_{22}=\frac{E_2}{1-\nu_1\nu_2}$;$B_{12}=\frac{\nu_2E_1}{1-\nu_1\nu_2}=\frac{\nu_1E_2}{1-\nu_1\nu_2}$, E_1, E_2 are main elasticity modules of the orthotropic material, *R* is radius of the middle surface of the shell, *h* is thickness of the shell, *u, v, w* are components of displacements of points of the middle surface of the shell, x_1, $x_2$are coordinates of curved edges of the shell; F_i, J_{zi}, J_{yi}, J_{kpi} are area and inertia moments of the cross-section of $i-th$ longitudinal bar in relation to the axis Oz and the axis parallel to the axis Oy and going through the gravity center of the cross section and even its inertia moment at torsion; F_c isarea of the cross section of the longitudinal rib; E_i, G_iareelasticity modulus and modulus of shear of the material of $i-th$ longitudinal bar, *t*-time coordinate, $t_1=\omega_0 t, \omega_0=\sqrt{\frac{E_1}{(1-\nu^2)\rho_0R^2}}$, ρ_0, ρ_i are density of the materials of the shell, $i-th$ is respectively a longitudinal bar.

The expressions for internal forces and moments are expressed in the following form:

$$N_{ij}=\int_{-h/2}^{h/2}\left(\sigma_{ij}+zw_{ij}\right)dz;\quad M_{ij}=-\int_{-h/2}^{h/2}\left(\sigma_{ij}+zw_{ij}\right)zdz \tag{1.2}$$

$$w_{11}=b_{11}\chi_{11}+b_{12}\chi_{22};\quad w_{22}=b_{12}\chi_{11}+b_{22}\chi_{22};\quad w_{21}=w_{12}=b_{66}\chi_{12}.$$

Stresses σ_{ij} and deformations ε_{ij} in the middle surface in (1.2) are determined in the following form:

$$\sigma_{11}=b_{11}\varepsilon_{11}+b_{12}\varepsilon_{22};\quad \sigma_{22}=b_{12}\varepsilon_{11}+b_{22}\varepsilon_{22};\quad \sigma_{12}=b_{66}\varepsilon_{12} \tag{1.3}$$

$$\varepsilon_{ij}=\bar{\varepsilon}_{ij}+\int_{-\infty}^{t}\Gamma\left(t-\tau\right)\bar{\varepsilon}_{ij}\left(\tau\right)d\tau \tag{1.4}$$

$$\bar{\varepsilon}_{11}=\frac{\partial u}{\partial x};\quad \bar{\varepsilon}_{22}=\frac{\partial v}{\partial y}+w;\ \bar{\varepsilon}_{12}=\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x};\ \chi_{11}=\frac{\partial^2 w}{\partial x^2};\chi_{22}=\frac{\partial^2 w}{\partial y^2};\chi_{12}=-2\frac{\partial^2 w}{\partial x\partial y};G\left(t\right)=Ae^{-\psi t};$$

The equations of motion of longitudinally reinforced and loaded with axially compressive forces orthotropic shell with flowing liquid are obtained on the base of Ostrograskii-Hamiliton's principle of stationarity of action:

$$\delta W=0 \tag{1.5}$$

where $W=\int_{t'}^{t''}Ldt-$ action by Hamilton, $L-$ Lagrangian function, $t^{'}$ and $t^{''}$ are given arbitrary moments of time.

Assuming that the main flow velocity is equal to U and deviations from this velocity are small we use the wave equation for the potential of perturbation velocities φ by [11]:

$$\Delta\varphi-\frac{1}{a_0^2}\left(\frac{\partial^2\varphi}{\partial t^2}+2U\frac{\partial^2\varphi}{R\partial\xi\partial t}+U^2\frac{\partial^2\varphi}{R^2\partial\xi^2}\right)=0 \tag{1.6}$$

On the contact surface shell-liquid the continuity of radial velocities and pressuresis observed. The condition of impermeability or flow smooth of the shell wall has the form [11, 6]:

$$\vartheta_r|_{r=R}=\frac{\partial\varphi}{\partial r}\bigg|_{r=R}=-\left(\omega_0\frac{\partial w}{\partial t_1}+U\frac{\partial w}{R\partial\xi}\right) \tag{1.7}$$

Equality of radial pressures from the liquid to the shell is

$$q_z = -p_{|r=R} \tag{1.8}$$

By complementing with contact conditions (1.7), (1.8), the expression for the total energy of the shell (1.1), equations of liquid motion (1.6) we get the problem on free oscillations of supported by longitudinal rib systems loaded with axial compressive forces of an orthotropic cylindrical shell with flowing liquid.

2 Solution of the problem

We will seek the displacements of the shell in the form:

$$\begin{aligned} u &= u_0 \sin\chi\xi \cos n\theta \sin\omega_1 t_1 \\ \vartheta &= \vartheta_0 \cos\chi\xi \sin n\theta \sin\omega_1 t_1 \\ w &= w_0 \cos\chi\xi \cos n\theta \sin\omega_1 t_1 \end{aligned} \tag{2.1}$$

Here u_0, ϑ_0, w_0−unknown constants; χ, n−wave numbers in the longitudinal and circumferential directions respectively.

The potential of perturbation velocities φ we seek in the form:

$$\varphi(\xi, r, \theta, t_1) = f(r) \cos n\varphi \sin kx \sin\omega t \tag{2.2}$$

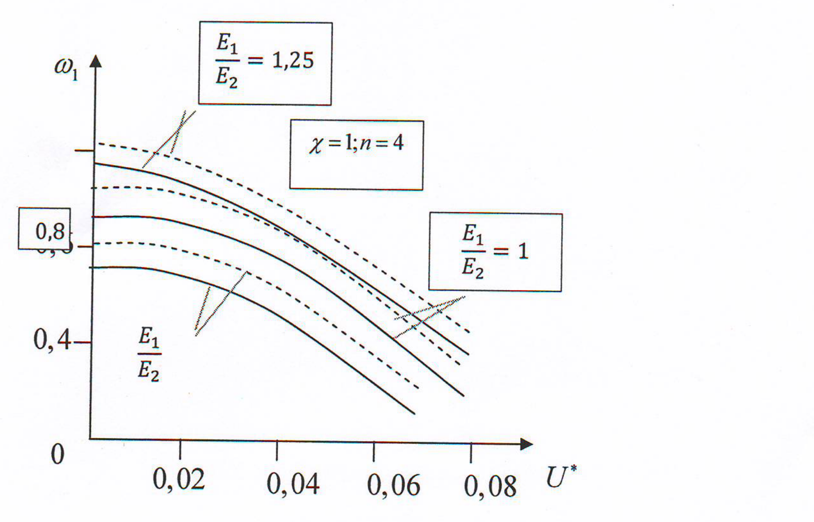

Fig 1. Dependence of the parameter of frequency on the flow velocity for the reinforced by longitudinal rib systems cylindrical shell with flowing liquid.

Using (2.2) from the conditions (1.7) and (1.6) we get:

$$\varphi = -\Phi_{\alpha n}\left(\omega_0 \frac{\partial w}{\partial t_1} + U\frac{\partial w}{R\partial\xi}\right) \tag{2.3}$$

$$p = \Phi_{\alpha n}\rho_m\left(\omega_0^2\frac{\partial^2 w}{\partial t_1^2} + 2U\omega_0\frac{\partial^2 w}{R\partial\xi\partial t_1} + U^2\frac{\partial^2 w}{R^2\partial\xi^2}\right),$$

where

$$\Phi_{\alpha n} = \begin{cases} I_n(\beta r)/I_n'(\beta r), & M_1 < 1 \\ J_n(\beta_1 r)/J_n'(\beta_1 r), & M_1 > 1 \\ \frac{R^n}{nR^{n-1}}, & M_1 = 1 \end{cases} \tag{2.4}$$

Here $M_1 = \frac{U+\omega_0 R\omega_1/\alpha}{a_0}$, $\beta^2 = R^{-2}\left(1 - M_1^2\right)\chi^2, \beta_1^2 = R^{-2}\left(M_1^2 - 1\right)\chi^2$, I_n is modified Bessel function of the first kind of order n, J_n are Bessel functions of the first kind of order n.

Further, in (1.8) the value $q_z = -p$ must be taken as q_z, where $p-$pressure in (2.3). Taking into account (2.1) the pressure p can be given as :

$$p = \frac{\rho_m \Phi_{\alpha n}}{\rho_0 \omega_0^2 h}\left(\omega_0^2\omega_1^2 + 2\omega_0\omega_1\chi U + \chi^2 U^2\right) w \tag{2.5}$$

Substituting (2.5) and (2.1) into (1.5) the problem is reduced to a homogeneous system of linear algebraic equations of the third order:

$$a_{i1}u_0 + a_{i2}v_0 + a_{i3}w_0 = 0 \qquad (i = 1, 2, 3) \tag{2.6}$$

Elements a_{i1}, a_{i2}, a_{i3} $(i = 1, 2, 3)$ have a huge form that is why we do not present them here. Nontrivial solution of the system of linear algebraic equations (2.6) of the third order is possible only in the case when ω_1 is the root of its determinant. Determination of ω_1 is reduced to a transcendental equation since ω_1 is included into the arguments of Bessel function J_n:

$$\begin{vmatrix} 2\left(\breve{\varphi}_{11} - \psi_{11}\omega_1^2\right) & \breve{\varphi}_{44} & \breve{\varphi}_{55} \\ \breve{\varphi}_{44} & 2\left(\breve{\varphi}_{22} - \psi_{22}\omega_1^2\right) & \breve{\varphi}_{66} \\ \breve{\varphi}_{55} & \breve{\varphi}_{66} & 2\left(\breve{\varphi}_{33} - \psi_{33}\omega_1^2 + l_1\sigma_x + q_z^{(0)}\psi_2\right) \end{vmatrix} = 0 \tag{2.7}$$

Note that for $U = 0\,(\varphi_2 = 0)$equations (2.7) goes over to the frequency equation of free oscillations of the supported by longitudinal rib systems, loaded withaxial compressive forces of the orthotropic cylindrical shell filled a liquid at rest. In the following we will consider some results of calculations provided by a computer program on the base of above-presented dependencies.

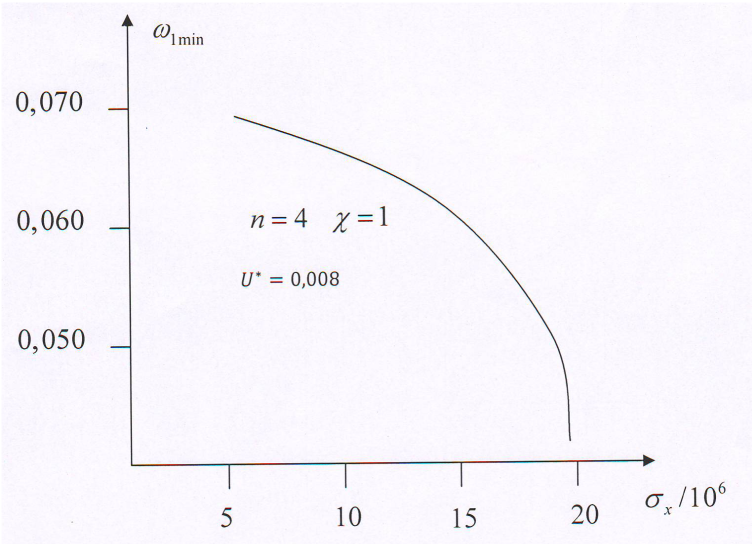

Fig. 2.Dependence of the parameter of frequency of the reinforced by longitudinal rib systems cylindrical shell with a flowing liquid on compressive stresses

For geometrical and physical parameters characterizing the materials of the shell and the mediumthe following ones were taken:

$$F_i = 3,4\,mm^2, \;\; J_{yi} = 5,1mm^4, \;\; \rho_0/\rho_m = 0,105. \;\; E_1 = 6,67 \cdot 10^9/m^2,$$

$$\rho_0 = \rho_i = 0,26 \cdot 10^4\,N^2/m^4, \;\; J_{yi} = 5,1mm^4, \;\; h_i = 1,39mm, \;\; A = 0,1615; \;\; \beta = 0,05;$$

$$B_{11} = 18,3GPa, \;\; B_{12} = 2,77GPa, B_{22} = 25,2GPa, \;\; h = 0,45mm,$$

$$\frac{I_{kp.i}}{2\pi R^3 h} = 0,5305 \cdot 10^{-6}; \;\; L = 800mm$$

In figure 1 the dependencies of the frequency parameter ω_1 on the relative velocity of the flow $U^* = U/c, \; c = \omega_0 R$ are illustrated for different values χ and n.

In the graphs by dashed lines the oscillations of the elastic cylindrical shell, longitudinally supported and loaded with axial compressive forces and which contain a flowing liquid, are marked, and by the continuous lines the oscillations of the longitudinally supported and loaded with axial compressive forces of viscoelastic cylindrical shell with flowing liquid, are marked. It is visible thatthe increase of velocity and taking into account the viscosity of shell material cause the decrease of frequency. It is important to note the values U^* for which the frequency of oscillations turns to zero. Obviously, here loss of stability of the shell takes place.

Finally, fig. 2 illustrates the effect/influence of compressive stresses on parameter ω_1 of oscillations of the supported by longitudinal rib systems of the cylindrical shell with a flowing liquid. It is seen that by increasing of compressive stresses the parameter ω_1 of oscillations of the considered system decreases first slowly but then sharply at some values of compressive stresses.

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References

1. Aliyev, F.F.: *Eigen oscillations in an infinite elastic medium of the longitudinally supported cylindrical shell with a flowing liquid*. The Ministry of education of the Republic of Azerbaijan. Mechanics and Mechanical Engineering. (1), 3-5 (2006).
2. Aliyev, F.F. *Eigen oscillations in an infinite elastic medium of supported by cross-sectional rib system of the cylindrical shell with a flowing liquid*. The Ministry of Education of The Republic of Azerbaijan. Mechanics and Mechanical Engineering. (2), 10-12 (2007).
3. Amiro, I.Y., Zarutskii, B.A., Polyakov, P.S.: Ribbed cylindrical shells. *iv, Nauk. Dumka*, 248 p. (1973).
4. Latifov, F.S., Seyfullayev, F.A.: *Asymptotic analysis of oscillation eigenfrequency of orthotropic cylindrical shells in infinite elastic medium filled with liquid*. Trans. Natl. Acad. Sci. Azerb. Ser. Phys.-Tech. Math. Sci.. **XVIV** (1), 227-230 (2004).
5. Latifov, F.S., Suleymanova, S.G.: *The problem on eigen oscillations of the supported by cross-sectional rib system and loaded with axial compressive forces of cylindrical shell filled with a medium*. Mechanics of machines, equipment and materials. International Scientific Journal. The United Institute of mechanical Engineering of The National Academy of Sciences of Belarus. Minsk, (1), 59-62 (2009).
6. Latifov, F.S. Oscillations of shells with an elastic and liquid medium. *Baku. Elm*. 164 p. (1999).
7. Pirmamedov, I.T.: *Parametrical oscillations of nonlinear and non-homogeneous by thickness viscoelastic cylindrical shell contacting with a viscoelastic medium in the case of friction*. Reports of The NAS of Azerbaijan. (2), 35-42 (2008).
8. Pirmamedov, I.T. *Calculation of parametric oscillations of a nonhomogeneous by thickness viscoelastic bar in a viscoelastic soil*. International Scientific Journal. The United Institute of Mechanical Engineering of NAS of Belarus. Minsk. (3) (8), 52-56 (2009).
9. Seyfullayev, F.A. *Asymptotic analysis of eigen frequencies of axisymmetrical oscialltions of orthotropic cylindrical shell in an infinite elastic medium filled with a liquid*. Mechanics and Mechanical engineering. (4), 33-34 (2004).
10. Suleymanova, S.G.: *Eigen oscillations of logitudinally supported and loaded with axial compresive forces cylindrical shell filled with a medium.* Proceedings of IMM of NAS of Azerbaijan. **XXVSS**, 135-140 (2007).
11. Volmir, S.A.: Shells in the stream of liquid and gas. Problems of aeroelasticity. *Moscow. Nauka*. 416 p. (1976).