

## Nonexistence of global solutions of the mixed problem for a system of nonlinear wave equations with $q$ -Laplacian operators

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**Abstract.** We study the mixed problem for the system of nonlinear  $q$ -Laplacian wave equations and investigate the nonexistence of the global solutions.

**Keywords.** System of Nonlinear Wave Equations, nonexistence of global solutions, linear damping term, initial-boundary value problem, Laplacian operator

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### 1 Introduction

In this paper we consider the nonlinear initial-boundary value problem

$$\begin{cases} u_{1tt} - \Delta_q u_1 + (-\Delta)^\alpha u_{1t} = f_1(u_1, u_2), \\ u_{2tt} - \Delta_q u_2 + (-\Delta)^\alpha u_{2t} = f_2(u_1, u_2), \end{cases} \quad (1.1)$$

$$u_j(t, x) = 0, \quad t > 0, \quad x \in \partial\Omega, \quad j = 1, 2, \quad (1.2)$$

$$u_j(0, x) = \varphi_j(x), \quad u_{jt}(0, x) = \psi_j(x), \quad x \in \Omega, \quad j = 1, 2. \quad (1.3)$$

Here  $\Omega$  is a bounded domain in  $\mathbb{R}^n$ ,  $n \geq 1$  with smooth boundary  $\partial\Omega$ ,  $t > 0$ ,  $x \in \Omega$ ,  $0 < \alpha \leq 1$ ,

$$f_1(u_1, u_2) = |u_1 + u_2|^{2\rho}(u_1 + u_2) + |u_1|^{\rho-1}|u_2|^{\rho+1}u_1;$$

$$f_2(u_1, u_2) = |u_1 + u_2|^{2\rho}(u_1 + u_2) + |u_1|^{\rho+1}|u_2|^{\rho-1}u_2;$$

$$\Delta_q u = \sum_{i=1}^{\infty} \frac{\partial}{\partial x_i} \left( \left| \frac{\partial u}{\partial x_i} \right|^q \frac{\partial u}{\partial x_i} \right), \quad \text{where } q \geq 2;$$

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$$(-\Delta)^\alpha u = \sum_{j=1}^{\infty} \lambda_j^\alpha (u, \varphi_j) \varphi_j,$$

where  $0 < \lambda_1 < \lambda_2 \leq \lambda_3 \leq \dots$ ,  $\varphi_1, \varphi_2, \varphi_3, \dots$  are the sequence of eigenvalues and eigenfunctions of  $-\Delta$  in  $H_0^1(\Omega)$ , respectively.

The norm in  $L_q(\Omega)$  is denoted by  $\|\cdot\|_q$  and in  $\overset{\circ}{W}_q^1(\Omega) = \{u : u \in W_q^1(\Omega)\}$  we use the norm

$$\|u\|_{q,1}^q = \sum_{j=1}^n \|u_{x_j}\|_q^q.$$

For the reader's convenience, we recall some of the basic properties of the operators used here. The degenerate operator  $\Delta_q u$  is bounded, monotone and hemicontinuous from

$\overset{\circ}{W}_q^1(\Omega)$  to  $W_{q'}^{-1}(\Omega)$ , where  $\frac{1}{q} + \frac{1}{q'} = 1$ .

We investigate the nonexistence of the global solutions of problem (1.1)-(1.3).

## 2 Preliminaries and main results

First, we make some preparations. Assume  $0 < \alpha \leq 1$  and the  $\rho, q$  satisfy the condition

$$\rho \geq \frac{q-1}{2}; \quad (2.1)$$

$$0 < \rho < +\infty \quad \text{for } n \leq q; \quad (2.2)$$

$$0 < \rho < \frac{nq}{2(n-q)-1}, \quad \text{for } n > q. \quad (2.3)$$

**Lemma 2.1** (Local existence) *Let the condition (2.1)-(2.3) hold, then for any initial data  $u_{j0} \in \overset{\circ}{W}_q^1(\Omega)$ ,  $u_{j1} \in L_2(\Omega)$ , if  $T$  is small enough, then there exists a solution  $(u_1, u_2)u$  of (1.1)-(1.3) which satisfies*

$$u_j \in L_\infty(0, T; \overset{\circ}{W}_q^1(\Omega)), \quad u'_j \in L_\infty(0, T; L_2(\Omega)) \cap L_2(0, T; D((-\Delta)^{\alpha/2})). \quad (2.4)$$

For the weak solution the following energetic identity is valid

$$E(t) + \frac{1}{2} \sum_{j=1}^2 \int_0^t \int_\Omega |u_j(t, x)|^2 dx = E(0), \quad (2.5)$$

where

$$E(t) = \frac{1}{2} \sum_{j=1}^2 \int_\Omega |u_{jt}(t, x)|^2 dx + \frac{1}{q} \sum_{j=1}^2 |\nabla u_j(t, x)|^q dx - G(u_1(t, x), u_2(t, x)), \quad (2.6)$$

$$G(u_1, u_2) = \frac{1}{2(p+1)} \int_\Omega |u_1(t, x) + u_2(t, x)|^{2(p+1)} dx + \frac{1}{p+1} \int_\Omega |u_1(t, x)u_2(t, x)|^{p+1} dx. \quad (2.7)$$

To prove the Lemma 2.1 we use the Galerkin method (see [8]).

In [1] is proved that in the case when conditions (2.2), (2.3) and

$$\rho < \frac{q-1}{2} \quad (2.8)$$

hold, then local solution defined by Lemma 2.1 is global. In this work we prove that for the global solvability condition (2.8) is principle.

Similar problems for the Klein-Gordon systems with the usual dissipations, i.e. when  $\alpha = 0$  are enough investigated (see [2,3]).

For the semilinear equation with  $p$ -laplacian and with fractal dissipation the close results are obtained in [4,5].

When  $q = 2$ , and the damping term is given by  $|u_t|^r u_t$ ,  $r \geq 0$ , many authors studied the existence and uniqueness of the global solution and the blow up of the solution (see [6,7]).

Our main results are expressed by the following theorem.

**Theorem 2.1** *Let the conditions (2.1)-(2.3) hold, for any initial data*

*$(\varphi_j, \psi_j) \in \overset{\circ}{W}_q^1(\Omega) \times L_2(\Omega)$ , if*

$$E(0) = \frac{1}{2} \sum_{j=1}^2 \int_{\Omega} |\psi_j(x)|^2 + \frac{1}{q} \sum_{j=1}^2 |\nabla \varphi_j(x)|^q dx - G(\varphi_1(x), \varphi_2(x)) < 0. \quad (2.9)$$

*Then exists  $T' > 0$  such that*

$$\lim_{t \rightarrow T'-0} \sum_{j=1}^2 \|u_j(t, \cdot)\|_{2(p+1)} = +\infty,$$

*where  $0 < T' \leq T_0$  ( $T_0$  can be seen in the proof).*

*Proof of Theorem 2.1.* Using (2.5) and (2.9), we have

$$E(t) \leq E(0) < 0. \quad (2.10)$$

Let

$$\begin{aligned} H(t) &= q(-E(t)) + \left(\frac{q}{2} + 1\right) \sum_{j=1}^2 \int_{\Omega} |u_{jt}|^2 dx \\ &+ \frac{1}{q} \sum_{j=1}^2 \int_{\Omega} |\nabla u_j|^q dx + \frac{2(p+1)-q}{2} G(u_1, u_2). \end{aligned} \quad (2.11)$$

Now, we introduce

$$F(t) = \frac{1}{2} \sum_{j=1}^2 \int_{\Omega} |u_j(t, x)|^2 dx$$

for any solution  $u$ , then differentiate  $F(t)$  with respect to  $t$  we have

$$F'(t) = \sum_{j=1}^2 \int_{\Omega} u_j(t, x) u_{jt}(t, x) dx. \quad (2.12)$$

The following Statement is valid (see [9]).

**Lemma 2.2** *There exists  $C_1 > 0$ ,  $C_2 > 0$  such that the inequality*

$$C_1 \sum_{j=1}^2 \|\nu_j\|_{2(p+1)}^{2(p+1)} \leq \int_{\Omega} G(\nu_1, \nu_2) dx \leq C_2 \sum_{j=1}^2 \|\nu_j\|_{2(p+1)}^{2(p+1)}$$

is valid for all  $\nu_1, \nu_2 \in L_{2(p+1)}(\Omega)$ .

From (2.5) follows that

$$\frac{1}{q} \sum_{j=1}^2 \|\nabla u_j(t, \cdot)\|_q^q \leq G(u_1(t, x), u_2(t, x)). \quad (2.13)$$

Considering Lemma 2.2 from this we get

$$\sum_{j=1}^2 \|\nabla u_j(t, \cdot)\|_q^q \leq qC_2 \sum_{j=1}^2 \|u_j(t, \cdot)\|_{2(p+1)}^{2(p+1)}. \quad (2.14)$$

Using (1.1), (1.2), (2.5), (2.6) and (2.12) we obtain that

$$\begin{aligned} F''(t) &= \sum_{j=1}^2 \int_{\Omega} |u_{jt}(t, x)|^2 dx - \sum_{j=1}^2 \int_{\Omega} |\nabla u_j(t, x)|^q dx + 2(p+1) \int_{\Omega} G(u_1(t, x), u_2(t, x)) dx \\ &+ \sum_{j=1}^2 \int_{\Omega} (-\Delta)^{\alpha} u_{jt}(t, x) u_j(t, x) dx = q(-E(t)) + \left(1 + \frac{q}{2}\right) \sum_{j=1}^2 \int_{\Omega} |u_{jt}(t, x)|^2 dx \\ &+ [2(p+1) - q] \int_{\Omega} G(u_1(t, x), u_2(t, x)) dx + \sum_{j=1}^2 \int_{\Omega} (-\Delta)^{\alpha} u_{jt}(t, x) u_j(t, x) dx. \end{aligned}$$

Due to (2.11) we have the inequality

$$\begin{aligned} F''(t) &\geq H(t) + \frac{2(p+1) - q}{2} \int_{\Omega} G(u_1(t, x), u_2(t, x)) dx \\ &+ \sum_{j=1}^2 \int_{\Omega} (-\Delta)^{\alpha} u_{jt}(t, x) u_j(t, x) dx. \end{aligned} \quad (2.15)$$

Then considering Lemma 2.2 again we obtain

$$\begin{aligned} F''(t) &\geq H(t) + \frac{2(p+1) - q}{2} C_1 \sum_{j=1}^2 \|u_j(t, \cdot)\|_{2(p+1)}^{2(p+1)} \\ &+ \sum_{j=1}^2 \int_{\Omega} (-\Delta)^{\alpha} u_{jt}(t, x) u_j(t, x) dx. \end{aligned} \quad (2.16)$$

By virtue of (2.1)-(2.3), we have

$$\left\| (-\Delta)^{\frac{\alpha}{2}} u_j(t, \cdot) \right\|_2 \leq C_3 \left\| \nabla u_j(t, \cdot) \right\|_q^q. \quad (2.17)$$

It follows from (2.5) that

$$G(u_1, u_2) \geq -E(t). \quad (2.18)$$

Due to lemma 2.2 there exists  $C_4 > 0$  that provides

$$C_4 \sum_{j=1}^2 \|u_j\|_{2(p+1)}^{2(p+1)} \geq \sum_{j=1}^2 \|\nabla u_j\|_q^q. \quad (2.19)$$

From (2.19) and (2.20) we get

$$C_5 \sum_{j=1}^2 \|u_j\|_{2(p+1)}^{2(p+1)} \geq -E(t), \quad (2.20)$$

where  $C_5 > 0$  is some constant. From the last follows

$$C_6 \sum_{j=1}^2 \|u_j\|_{2(p+1)} \geq (-E(t))^{\frac{1}{2(p+1)}}, \quad (2.21)$$

$$G(u_1, u_2) \geq C_1 \sum_{j=1}^2 \|\nabla u_j\|_q^q \geq C_7 \left( \sum_{j=1}^2 \|\nabla u_j\|_q \right)^q, \quad (2.22)$$

where  $C_6 > 0, C_7 > 0$  are some constants.

By virtue of (2.3)  $1 < 2(p+1) < \frac{nq}{n-q}$ . So  $W_{q,0}^1(\Omega) \subset L_{2(p+1)}(\Omega)$ , i.e.

$$\|\nu\|_{2(p+1)} \leq C_8 \|\nabla \nu\|_q \quad (\text{see [10]}). \quad (2.23)$$

Therefore from (2.21) and (2.22) we get the inequality

$$C_6 C_8 \sum_{j=1}^2 \|\nabla u_j\|_q \geq (-E(t))^{\frac{1}{2(p+1)}}. \quad (2.24)$$

By virtue of (2.1)-(2.3)  $\frac{2-q}{2} \leq 0$ , therefore from (2.24) we get

$$C_9 \left( \sum_{j=1}^2 \|\nabla u_j\|_q \right)^{\frac{2-q}{2}} \leq (-E(t))^{\frac{2-q}{4(p+1)}}, \quad (2.25)$$

where  $C_9 = (C_6 C_8)^{\frac{2-q}{2}}$ .

Denoting  $\beta = \frac{2-q}{4(p+1)} + 1 = \frac{4p+6-q}{4(p+1)}$  we have  $-(1-\beta) = \frac{2-q}{4(p+1)}$ .

Further, using embedding theorems and taking into account conditions (2.1)-(2.3), (2.10) and (2.25), we obtain the following

$$\begin{aligned}
\left| \sum_{j=1}^2 \int_{\Omega} (-\Delta)^{\alpha} u_{jt}(t, x) u_j(t, x) dx \right| &= \sum_{j=1}^2 \left\| (-\Delta)^{\frac{\alpha}{2}} u_{jt} \right\|_2 \cdot \left\| (-\Delta)^{\frac{\alpha}{2}} u_j \right\|_2 \\
&\leq \sum_{j=1}^2 \left\| (-\Delta)^{\frac{\alpha}{2}} u_{jt} \right\|_2 \cdot \sum_{j=1}^2 \left\| (-\Delta)^{\frac{\alpha}{2}} u_j \right\|_2 \\
&\leq C_{10} \sum_{j=1}^2 \left\| (-\Delta)^{\frac{\alpha}{2}} u_{jt} \right\|_2 \cdot \sum_{j=1}^2 \left\| \nabla u_j \right\|_q \\
&= C_{10} \left\| (-\Delta)^{\frac{\alpha}{2}} u_{jt} \right\|_2 \cdot \left( \sum_{j=1}^2 \left\| \nabla u_j \right\|_q \right)^{\frac{q}{2}} \left( \sum_{j=1}^2 \left\| \nabla u_j \right\|_q \right)^{1-\frac{q}{2}} \\
&\leq C_{10} \left[ \frac{1}{2\varepsilon} \sum_{j=1}^2 \left\| (-\Delta)^{\frac{\alpha}{2}} u_j \right\|_2^2 + 2\varepsilon \sum_{j=1}^2 \left\| \nabla u_j \right\|_q^q \right] \cdot \left( \sum_{j=1}^2 \left\| \nabla u_j \right\|_q \right)^{1-\frac{q}{2}} \\
&\leq \frac{C_9 C_{10}}{2\varepsilon} (-E(t))^{-(1-\beta)} \sum_{j=1}^2 \left\| (-\Delta)^{\frac{\alpha}{2}} u_{jt} \right\|_2^2 \\
&\quad + 2\varepsilon C_9 C_{10} \left( \sum_{j=1}^2 \left\| \nabla u_j \right\|_q \right)^q (-E(0))^{-(1-\beta)}. \tag{2.26}
\end{aligned}$$

Setting  $\varepsilon = \frac{(2(p+1)-q)(-E(0))^{1-\beta}}{4C_9 C_{10}}$  we get

$$F''(t) \geq H(t) - C_9 C_{10} (-E(t))^{-(1-\beta)} \sum_{j=1}^2 \left\| (-\Delta)^{\frac{\alpha}{2}} u_{jt} \right\|_2^2. \tag{2.27}$$

Introduce the functional

$$\Phi(t) = (-E(t))^{\beta} + \beta \frac{2\varepsilon}{C_9 C_{10}} F'(t). \tag{2.28}$$

Using (2.27) and (2.28) we obtain

$$\begin{aligned}
\Phi'(t) &= \beta (-E(t))^{-(1-\beta)} (-E'(t)) + \beta \frac{2\varepsilon}{C_9 C_{10}} F''(t) \geq \beta (-E(t))^{-(1-\beta)} (-E'(t)) \\
&\quad + \beta \frac{2\varepsilon}{C_9 C_{10}} \left[ H(t) - \frac{C_9 C_{10}}{2\varepsilon} (-E(t))^{-(1-\beta)} \sum_{j=1}^2 \left\| (-\Delta)^{\frac{\alpha}{2}} u_{jt} \right\|_2^2 \right] = \frac{2\beta\varepsilon}{C_9 C_{10}} H(t) > 0. \tag{2.29}
\end{aligned}$$

Assume (2.9) holds, then

$$\Phi(0) = (-E(0))^{\beta} + \beta \frac{2\varepsilon}{C_9 C_{10}} \sum_{j=1}^2 \int_{\Omega} \varphi_j(x) \psi_j(x) dx > 0. \tag{2.30}$$

By (2.29) and (2.30), we have

$$\Phi(t) > \Phi(0) > 0.$$

Considering this from (2.28) it is easy to get

$$\Phi^{1/\beta}(t) \leq 4 \left\{ (-E(t)) + \left( \beta \frac{2\varepsilon}{C_9 C_{10}} F'(t) \right)^{1/\beta} \right\}. \quad (2.31)$$

From other side

$$\begin{aligned} |F'(t)| &\leq \sum_{j=1}^2 \left\| u_{jt} \right\|_{\frac{2(p+1)}{2p+1}} \left\| u_j \right\|_{2(p+1)} \\ &\leq \sum_{j=1}^2 \|u_{jt}\|_2 \|u_j\|_{2(p+1)} \leq \sum_{j=1}^2 \left\| u_{jt} \right\|_2 \sum_{j=1}^2 \left\| u_j \right\|_{2(p+1)}. \end{aligned} \quad (2.32)$$

Taking into account (2.26) and (2.32), from (2.31) we obtain

$$\begin{aligned} \Phi^{1/\beta}(t) &\leq 4 \left\{ (-E(t)) + \left( \beta \frac{2\varepsilon}{C_9 C_{10}} F'(t) \right)^{1/\beta} \right\} \\ &\leq 4 \left\{ (-E(t)) + \left( \beta \frac{2\varepsilon}{C_9 C_{10}} \sum_{j=1}^2 \left\| u_{jt} \right\|_2 \sum_{j=1}^2 \left\| u_j \right\|_{2(p+1)} \right)^{1/\beta} \right\} \\ &\leq 4 \left\{ (-E(t)) + \frac{1}{2} \left( \sum_{j=1}^2 \left\| u_{jt} \right\|_2 \right)^2 + \left( \beta \frac{2\sqrt{2}\varepsilon}{C_9 C_{10}} \sum_{j=1}^2 \left\| u_j \right\|_{2(p+1)} \right)^{\frac{2}{2\beta-1}} \right\} \\ &\leq 8 \left\{ (-E(t)) + \frac{1}{2} \sum_{j=1}^2 \int_{\Omega} |u_{jt}(t, x)|^2 dx + \left( \beta \frac{2\sqrt{2}\varepsilon}{C_9 C_{10}} \sum_{j=1}^2 \left\| u_j \right\|_{2(p+1)} \right)^{\frac{2}{2\beta-1}} \right\} \end{aligned}$$

By virtue of Lemma 2.2 and (2.11)

$$C_2 \frac{2(p+1) - q}{2} \sum_{j=1}^2 \|u_j\|_{2(p+1)}^{2(p+1)} \leq H(t).$$

Therefore

$$\Phi^{1/\beta}(t) \leq 8 \left\{ (-E(t)) + \frac{1}{2} \sum_{j=1}^2 \int_{\Omega} |u_{jt}(t, x)|^2 dx + \left( \beta \frac{2\sqrt{2}\varepsilon}{C_9 C_{10}} \sum_{j=1}^2 \left\| u_j \right\|_{2(p+1)} \right)^{\frac{2}{2\beta-1}} \right\}. \quad (2.33)$$

From (2.9) and (2.9) we get

$$C_{12} \sum_{j=1}^2 \left\| u_j \right\|_{2(p+1)} \geq -E(0).$$

It follows from here that

$$(-C_{12}E(0))^{2(p+1) - \frac{2}{2\beta-1}} \sum_{j=1}^2 \left\| u_j \right\|_{2(p+1)}^{2(p+1) - \frac{2}{2\beta-1}} \geq 1.$$

Consideration this in (2.33) gives

$$\begin{aligned} \Phi^{1/\beta}(t) &\leq 8 \left\{ (-E(t)) + \frac{1}{2} \sum_{j=1}^2 \int_{\Omega} |u_{jt}(t, x)|^2 dx \right. \\ &\quad \left. + \left( \beta \frac{2\varepsilon}{C_9 C_{10}} \sum_{j=1}^2 \|u_j\|_{2(p+1)} \right)^{\frac{2}{2\beta-1}} (-C_{12} E(0))^{2(p+1) - \frac{2}{2\beta-1}} \sum_{j=1}^2 \|u_j\|_{2(p+1)}^{2(p+1) - \frac{2}{2\beta-1}} \right\} \\ &\leq C_{13} H(t). \end{aligned} \quad (2.34)$$

Using (2.29) and (2.34) we get

$$\frac{d}{dt} \Phi^{1-\frac{1}{\beta}}(t) = \left(1 - \frac{1}{\beta}\right) \Phi^{-\frac{1}{\beta}}(t) \Phi'(t) \leq -C_{14},$$

where  $C_{14} = \frac{2\varepsilon(1-\beta)}{C_9 C_{10} C_{13}} > 0$ . Therefore we have

$$\Phi(t) \geq \left[ \frac{\Phi^{\frac{1-\beta}{\beta}}(0)}{1 - C_{14} \Phi^{\frac{1-\beta}{\beta}}(0) \cdot t} \right]^{\frac{\beta}{1-\beta}}.$$

Hence there exists a  $T' > 0$  such that

$$T' \leq T_0 = \frac{1}{C_{14} \Phi^{\frac{1-\beta}{\beta}}(0)}$$

and

$$\lim_{t \rightarrow T'^{-0}} \Phi(t) = +\infty. \quad (2.35)$$

By virtue of the energetic inequality

$$(-E(t)) + \frac{1}{2} \sum_{j=1}^2 \int_{\Omega} |u_{jt}(t, x)|^2 dx = \frac{1}{q} \sum_{j=1}^2 |\nabla u_j(t, x)|^q dx - \int_{\Omega} G(u_1(t, x), u_2(t, x)) dx.$$

From the last

$$\begin{aligned} (-E(t)) + \frac{1}{2} \sum_{j=1}^2 \int_{\Omega} |u_{jt}(t, x)|^2 dx &\leq \int_{\Omega} G(u_1(t, x), u_2(t, x)) dx \\ &\leq C_2 \sum_{j=1}^2 \|u_j\|_{2(p+1)}^{2(p+1)}. \end{aligned} \quad (2.36)$$

It follows from (2.35) and (2.36) that

$$\lim_{t \rightarrow T'^{-0}} \sum_{j=1}^2 \|u_j\|_{2(p+1)} = +\infty.$$



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