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Some fixed- point type theorems on parametric soft $b-$metric spaces

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Abstract. *A generalization of classical metric, namely parametric metric was introduced by Hussain et al. in [14]. The significance of investigate this construction firstly take care of metric conditions and secondly, chance of performing some kind of metric space namely parametric soft $b-$metric space to training of fixed point type theorems. Because of this, in this paper we present new impression in this construction and set up varied fixed point theorems using continuous and surjective mappings in this space.*

Keywords. parametric $b-$metric · parametric soft $b-$ metric space · soft continuous mapping · fixed point theorem

Mathematics Subject Classification (2010): 54H25, 47H10

1 Introduction and preliminaries

The concept of metric is one of the most structures of functional analysis, real analysis and topology. Therefore, many researchers have been very curious about this field and have defined different types of metrics. In the literature, many studies have been carried out on the generalization of the metric structure, for instance $G-$metric [21], $S-$metric [22], $2-$metric [12], $D-$metric [11], dislocated metric [16], cone metric [17], $b-$metric [5,6], parametric metric [14], bipolar metric [8], parametric $S-$metric [23], parametric $b-$metric [15] etc. It is known that, although the concept of $b-$metric seems to come to the fore with Czerwik's interesting approaches, it was actually beforehand handled by some researchers, e.g. Bourbaki [4], Berinde [2] and Heinhonen [7] etc. Recently, many researchers have studied generalized metric space by changing the triangle inequality of metric conditions. Soft set theory was introduced by Molodtsov [20] as a new mathematical structure. Since applications of soft set theory in other disciplines and real life problems was progressing rapidly, the study of soft metric space which is based on soft point of soft sets was initiated

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by Das and Samanta [10]. Yazar et al. [25] examined some important properties of soft metric spaces and soft continuous mappings. They also proved some fixed point theorems of soft contractive mappings on soft metric spaces. A number of authors introduced contractive type mapping on a complete metric space which are generalizations of Banach contraction, and which have the property that each such mapping has a unique fixed point in ([18],[24]). By using contractive type mapping, some authors have studied the fixed point theory for soft functions on different soft metric spaces which are generalizations of metric spaces such as soft $S-$metric space [13], bipolar soft metric space [9] etc.

A generalization of classical metric, namely parametric metric was introduced by Hussain et al. in [14]. In this paper, we will deal with the parametric soft $b-$metric structure, which is one of the most remarkable generalizations of the metric. The significance of investigate this construction firstly take care of metric conditions and secondly, chance of performing some kind of metric space namely parametric soft $b-$metric space to training of fixed point type theorems. Because of this, in this paper we present new impression in this construction and set up varied fixed point theorems using contractive mappings in this space. Initially, we acquaint the notions of parametric metric, parametric soft metric , parametric $S-$metric, parametric $b-$metric and soft $S-$metric. Later we successfully give parametric soft $b-$metric space using the definitions of parametric $b-$metric and parametric soft metric. It is known that contractive mappings has a major area in the fixed point theory. We close this work some kind of interesting fixed point theorems and demonstrate that when contractive mappings defined on a complete parametric soft $b-$metric space has a unique fixed soft point.

Throughout this paper, X denotes initial universe, E denotes the set of all parameters, $P(X)$ denotes the power set of X.

Definition 1.1 *[20] A pair F_E is called a soft set over X, where F is a mapping given by $F : E \to P(X)$.*

Definition 1.2 *[19] If for all $a \in E$, $F(a) = \varnothing$, F_E is said to be a null soft set denoted by Φ.If for all $a \in E$, $F(a) = X$, then F_E is said to be an absolute soft set denoted by $\widetilde{X}$.*

Definition 1.3 *([1],[10]) Let F_E be a soft set over X. The soft set F_E is called a soft point, denoted by x_{a_E}, if for the element $a \in E$, $F(a) = \{x\}$ and $F(a') = \varnothing$ for all $a' \in E - \{a\}$ (briefly denoted by x_a) .*

To give the family of all soft sets on X it is sufficient to give only soft points on X. So each soft set can be expressed as a union of soft points

Definition 1.4 *[1] The soft point x_a is said to be belonging to the soft set F_E, denoted by $x_a \widetilde{\in} F_E$, if $x_a(a) \in F(a)$,i.e., $\{x\} \subseteq F(a)$.*

Definition 1.5 *[10] Let $\mathbb{R}$ be the set of all real numbers, $B(\mathbb{R})$ be the collection of all non-empty bounded subsets of $\mathbb{R}$ and E be taken as a set of parameters. Then a mapping $F : E \to B(\mathbb{R})$ is called a soft real set. If F_E is a singleton soft set, then it will be called a soft real number and denoted by $\widetilde{r}, \widetilde{s}, \widetilde{t}$ etc. Here $\widetilde{r}, \widetilde{s}, \widetilde{t}$ will denote a particular type of soft real numbers such that $\widetilde{r}(a) = r$, for all $a \in E$. $\widetilde{0}$ and $\widetilde{1}$ are the soft real numbers where $\widetilde{0}(a) = 0$, $\widetilde{1}(a) = 1$ for all $a \in E$, respectively.*

Definition 1.6 *[14] Let X be a nonempty set and $P : X \times X \times (0, \infty) \to [0, \infty)$ be a function satisfing the following conditions for all $x, y, a \in X$ and $t > 0$,*

(1) $P(x, y, t) = 0$ if and only if $x = y$,
(2) $P(x, y, t) = P(y, x, t)$,
(3) $P(x, y, t) \leq P(x, a, t) + P(a, y, t)$.

Then P is called a parametric metric on X and the pair (X, P) is called a parametric metric space.

Definition 1.7 *[3] A parametric soft metric on X is a mapping $S_p : SP(\widetilde{X}) \times SP(\widetilde{X}) \times \mathbb{R}(E)^* \to \mathbb{R}(E)^*$ that satisfies the following conditions, for each soft points $x_a, y_b, z_c \in SP(\widetilde{X})$ and all $\widetilde{t} > \widetilde{0}$,*

(1) $S_p\left(x_a, y_b, \widetilde{t}\right) = \widetilde{0}$ if and only if $x_a = y_b$,
(2) $S_p\left(x_a, y_b, \widetilde{t}\right) = S_p\left(y_b, x, \widetilde{t}\right)$,
(3) $S_p\left(x_a, y_b, \widetilde{t}\right) \leq S_p\left(x_a, z_c, \widetilde{t}\right) + S_p\left(z_c, y_b, \widetilde{t}\right)$.
Then S_p is called a parametric soft metric on X and the pair (X, S_p, E) is called a parametric soft metric space.

Definition 1.8 *[23] Let X be a nonempty set and $S : X^3 \times (0, \infty) \to [0, \infty)$ be a function satisfing the following conditions for all $x, y, z, a \in X$ and $t > 0$,*

(1) $S(x, y, z, t) = 0$ if and only if $x = y = z$,
(2) $S(x, y, z, t) \leq S(x, x, a, t) + S(y, y, a, t) + S(z, z, a, t)$.
Then S is called a parametric $S-$metric on X and the pair (X, S) is called a parametric $S-$metric space.

Definition 1.9 *[13] A soft $S-$metric on X is a mapping $S : SP(\widetilde{X}) \times SP(\widetilde{X}) \times SP(\widetilde{X}) \to \mathbb{R}(E)^*$ that satisfies the following conditions, for each soft points $x_a, y_b, z_c, u_d \in SP(\widetilde{X})$,*

S1) $S(x_a, y_b, z_c) \widetilde{\geq} \widetilde{0}$,
S2) $S(x_a, y_b, z_c) = \widetilde{0}$ if and only if $x_a = y_b = z_c$,
S3) $S(x_a, y_b, z_c) \widetilde{\leq} S(x_a, x_a, u_d) + S(y_b, y_b, u_d) + S(z_c, z_c, u_d)$.
Then the soft set $\widetilde{X}$ with a soft $S-$metric S is called a soft $S-$metric space and denoted by $\left(\widetilde{X}, S, E\right)$.

2 Parametric Soft $b-$Metric Spaces

In next section, we firstly introduce the concepts of parametric soft $b-$metric space and parametric soft $S-$metric space. Also, we investigate some relationships between parametric soft metric, parametric soft $S-$metric and parametric soft $b-$metric. Later we give the existence and uniqueness of some fixed soft points of continuous and surjective mapping satisfying contractive condition in complete parametric soft $b-$metric space. Let $\widetilde{X}$ be the absolute soft set, E be a non-empty set of parameters and $SP(\widetilde{X})$ be the collection of all soft points of $\widetilde{X}$. Let $\mathbb{R}(E)^*$ denote the set of all non-negative soft real numbers.

Definition 2.1 *A parametric soft $S-$metric on $SP(\widetilde{X})$ is a mapping $d_S : SP(\widetilde{X}) \times SP(\widetilde{X}) \times SP(\widetilde{X}) \times \mathbb{R}(E)^* \to \mathbb{R}(E)^*$ that satisfies the following conditions:*

S1) $d_S\left(x_a, y_b, z_c, \widetilde{t}\right) \widetilde{\geq} \widetilde{0}$,
S2) $d_S\left(x_a, y_b, z_c, \widetilde{t}\right) = \widetilde{0}$ if and only if $x_a = y_b = z_c$,
S3) $d_S\left(x_a, y_b, z_c, \widetilde{t}\right) \widetilde{\leq} d_S\left(x_a, x_a, u_d, \widetilde{t}\right) + d_S\left(y_b, y_b, u_d, \widetilde{t}\right) + d_S\left(z_c, z_c, u_d, \widetilde{t}\right)$
for each soft points $x_a, y_b, z_c, u_d \in SP(\widetilde{X})$ and all $\widetilde{t} > \widetilde{0}$.
Then the soft set $\widetilde{X}$ with a parametric soft $S-$metric d_S is called a parametric soft $S-$metric space and denoted by $\left(\widetilde{X}, d_S, E\right)$.

Lemma 2.1 *Let* $\left(\widetilde{X}, d_S, E\right)$ *be a parametric soft* $S-$*metric space. Then*

$$d_S\left(x_a, x_a, y_b, \widetilde{t}\right) = d_S\left(y_b, y_b, x_a, \widetilde{t}\right)$$

for all $x_a, y_b \in SP(\widetilde{X})$ *and* $\widetilde{t} > \widetilde{0}$.

Proof. For all $x_a, y_b \in SP(\widetilde{X})$ and $\widetilde{t} > \widetilde{0}$, by using condition of parametric soft $S-$metric, we have

$$\begin{aligned} d_S\left(x_a, x_a, y_b, \widetilde{t}\right) &\leq 2d_S\left(x_a, x_a, x_a, \widetilde{t}\right) + d_S\left(y_b, y_b, x_a, \widetilde{t}\right) \\ &= d_S\left(y_b, y_b, x_a, \widetilde{t}\right), \qquad (2.1) \\ d_S\left(y_b, y_b, x_a, \widetilde{t}\right) &\leq 2d_S\left(y_b, y_b, y_b, \widetilde{t}\right) + d_S\left(x_a, x_a, y_b, \widetilde{t}\right) \\ &= d_S\left(x_a, x_a, y_b, \widetilde{t}\right). \qquad (2.2) \end{aligned}$$

From the inequalities (2.1) and (2.2), $d_S\left(x_a, x_a, y_b, \widetilde{t}\right) = d_S\left(y_b, y_b, x_a, \widetilde{t}\right)$ is satisfied.

Definition 2.2 *A parametric soft* $b-$*metric on* $SP(\widetilde{X})$ *is a mapping* $b_S : SP(\widetilde{X}) \times SP(\widetilde{X}) \times \mathbb{R}(E)^* \to \mathbb{R}(E)^*$ *that satisfies the following conditions:*

b1) $b_S\left(x_a, y_b, \widetilde{t}\right) = \widetilde{0}$ if and only if $x_a = y_b$,
b2) $b_S\left(x_a, y_b, \widetilde{t}\right) = b_S\left(y_b, x_a, \widetilde{t}\right)$,
b3) $b_S\left(x_a, y_b, \widetilde{t}\right) \widetilde{\leq} \widetilde{s}\left[b_S\left(x_a, z_c, \widetilde{t}\right) + b_S\left(z_c, y_b, \widetilde{t}\right)\right]$
for each soft points $x_a, y_b, z_c \in SP(\widetilde{X})$ and all $\widetilde{t} > \widetilde{0}, \widetilde{s} \geq \widetilde{1}$.

Then the soft set $\widetilde{X}$ with a parametric soft $b-$metric b_S is called a parametric soft $b-$metric space and denoted by $\left(\widetilde{X}, b_S, \widetilde{s}, E\right)$.

Remark 2.1 If $\widetilde{s} = \widetilde{1}$, parametric soft $b-$metric is a parametric soft metric.

Definition 2.3 *Let* $\left(\widetilde{X}, b_S, \widetilde{s}, E\right)$ *be a parametric soft* $b-$*metric space and* $\left\{x_{a_n}^n\right\}$ *be a soft sequence of soft points in* $\widetilde{X}$.

(i) The soft sequence $\left\{x_{a_n}^n\right\}$ is called convergent to x_a, written as $\lim_{n\to\infty} x_{a_n}^n = x_a$, if $\lim_{n\to\infty} b_S\left(x_{a_n}^n, x_a, \widetilde{t}\right) = \widetilde{0}$ for all $\widetilde{t} \widetilde{>} \widetilde{0}$.

(ii) The soft sequence $\left\{x_{a_n}^n\right\}$ is called a Cauchy sequence if $\lim_{n,m\to\infty} d_S\left(x_{a_n}^n, x_{a_m}^m, \widetilde{t}\right) = \widetilde{0}$ for all $\widetilde{t} \widetilde{>} \widetilde{0}$.

(iii) A parametric soft $b-$metric $\left(\widetilde{X}, b_S, \widetilde{s}, E\right)$ is called complete if every Cauchy sequence is convergent.

Example 2.1. Let $E = \mathbb{R}$ be a parameter set and $X = [0, \infty)$, $\widetilde{t} > \widetilde{0}$. Consider usual metrics on this sets and define

$$b_S : SP(\widetilde{X}) \times SP(\widetilde{X}) \times \mathbb{R}(E)^* \to \mathbb{R}(E)^*$$

is defined by

$$b_s\left(x_a, y_b, \widetilde{t}\right) = \widetilde{2}\widetilde{t}\left(|a-b| + |x-y|\right).$$

Then it can be easily verified that b_S is a parametric soft $b-$metric space with constant $\widetilde{2}$ on $SP(\widetilde{X})$.

Definition 2.4 *Let* $\left(\widetilde{X}, b_S, \widetilde{s}, E\right)$ *be a parametric soft* $b-$*metric space and* $f_\varphi : \left(\widetilde{X}, b_S, \widetilde{s}, E\right) \to \left(\widetilde{X}, b_S, \widetilde{s}, E\right)$ *be a soft mapping. Then* f_φ *is a soft continuous mapping at soft point* x_a *in* $\widetilde{X}$, *if for any soft sequence* $\left\{x_{a_n}^n\right\}$ *in* $\widetilde{X}$ *such that* $\lim_{n\to\infty} b_S\left(x_{a_n}^n, x_a, \widetilde{t}\right) = \widetilde{0}$, *then* $\lim_{n\to\infty} b_S\left(f_\varphi\left(x_{a_n}^n\right), f_\varphi\left(x_a\right), \widetilde{t}\right) = \widetilde{0}$ *is satisfied.*

Lemma 2.2 *Let* $(X, b_S, \widetilde{s}, E)$ *be a parametric soft* $b-$*metric space with the coefficient* $\widetilde{s} = \widetilde{1}$. *Let* $\left\{x_{a_n}^n\right\}$ *be a soft sequence of soft points in* $\widetilde{X}$ *such that*

$$b_S\left(x_{a_n}^n, x_{a_{n+1}}^{n+1}, \widetilde{t}\right) \leq \widetilde{\vartheta} b_S\left(x_{a_{n-1}}^{n-1}, x_{a_n}^n, \widetilde{t}\right),$$

where $\widetilde{0} \leq \widetilde{\vartheta} < \frac{\widetilde{1}}{\widetilde{s}}$, $n = 1, 2, ...$*Then* $\left\{x_{a_n}^n\right\}$ *is a Cauch sequence in* $(X, b_S, \widetilde{s}, E)$.

Proof. It is clear from the definition of parametric soft $b-$metric.

Remark 2.2 By using parametric soft $S-$metric, we obtain parametric soft $b-$metric.

Lemma 2.3 *Let* $\left(\widetilde{X}, d_S, E\right)$ *be a parametric soft* $S-$*metric space and let the function*

$$b_S : SP(\widetilde{X}) \times SP(\widetilde{X}) \times \mathbb{R}(E)^* \to \mathbb{R}(E)^*$$

be defined by

$$b_s\left(x_a, y_b, \widetilde{t}\right) = d_S\left(x_a, x_a, y_b, \widetilde{t}\right),$$

for each $x_a, y_b \in SP(\widetilde{X})$ *and* $\widetilde{t} > \widetilde{0}$. *Then* b_S *is a parametric soft* $b-$*metric.*

Proof. By using S1), we have the conditions b1) and b2). Now we show that b3) is obtained. Using condition and Lemma 2.1, we have

$$\begin{aligned} b_s\left(x_a, y_b, \widetilde{t}\right) &= d_S\left(x_a, x_a, y_b, \widetilde{t}\right) \\ &\leq d_S\left(x_a, x_a, z_c, \widetilde{t}\right) + d_S\left(x_a, x_a, z_c, \widetilde{t}\right) + d_S\left(y_b, y_b, z_c, \widetilde{t}\right) \\ &= 2d_S\left(x_a, x_a, z_c, \widetilde{t}\right) + d_S\left(y_b, y_b, z_c, \widetilde{t}\right) \\ &= 2b_s\left(x_a, z_c, \widetilde{t}\right) + b_s\left(y_b, z_c, \widetilde{t}\right) \\ &\leq 2\left[b_s\left(x_a, z_c, \widetilde{t}\right) + b_s\left(z_c, y_b, \widetilde{t}\right)\right]. \end{aligned}$$

Lemma 2.4 *Let* $\left(\widetilde{X}, S_p, E\right)$ *be a parametric soft metric space and let the function*

$$d_S^{S_p} : SP(\widetilde{X}) \times SP(\widetilde{X}) \times SP(\widetilde{X}) \times \mathbb{R}(E)^* \to \mathbb{R}(E)^*$$

be defined by

$$d_S^{S_p}\left(x_a, y_b, z_c, \widetilde{t}\right) = S_p\left(x_a, z_c, \widetilde{t}\right) + S_p\left(y_b, z_c, \widetilde{t}\right)$$

for each $x_a, y_b, z_c \in SP(\widetilde{X})$ *and* $\widetilde{t} > \widetilde{0}$. *Then* $d_S^{S_p}$ *is a parametric soft* $S-$*metric.*

Proof. It can be immediately taken from definition of parametric soft metric and parametric soft $S-$metric.

3 Some Fixed- Point Theorems in Parametric Soft $b-$Metric Spaces

In tis section, we prove the existence and uniqueness of some fixed soft points of continuous and surjective mapping satisfying contractive condition for important fixed-point theorems in complete parametric soft $b-$metric space.

Theorem 3.1 *Let* $\left(\widetilde{X}, b_S, \widetilde{s}, E\right)$ *be a complete parametric soft* $b-$*metric space and* $f_\varphi : \left(\widetilde{X}, b_S, \widetilde{s}, E\right) \to \left(\widetilde{X}, b_S, \widetilde{s}, E\right)$ *be a continuous and surjective mapping satisfying the following condition*

$$b_S\left(f_\varphi(x_a), f_\varphi(y_b), \widetilde{t}\right) + \widetilde{\alpha}\max\left\{b_S\left(x_a, f_\varphi(y_b), \widetilde{t}\right), b_S\left(y_b, f_\varphi(x_a), \widetilde{t}\right)\right\}$$
$$\geq \widetilde{\beta}\frac{b_S\left(x_a, f_\varphi(x_a), \widetilde{t}\right)\left[\widetilde{1} + b_s\left(y_b, f_\varphi(y_b), \widetilde{t}\right)\right]}{\widetilde{1} + b_s\left(x_a, y_b, \widetilde{t}\right)} + \widetilde{\gamma} b_s\left(x_a, y_b, \widetilde{t}\right) \tag{3.1}$$

for all $x_a, y_b \in SP(\widetilde{X})$, $x_a \neq y_b$ and for all $\widetilde{t} > \widetilde{0}$, where $\widetilde{\alpha}, \widetilde{\beta}, \widetilde{\gamma} \geq \widetilde{0}$ are soft real constants and $\widetilde{s}\widetilde{\beta} + \widetilde{\gamma} > \left(\widetilde{1} + \widetilde{\alpha}\right)\widetilde{s} + \widetilde{s}^2\widetilde{\alpha}$, $\widetilde{\gamma} > \left(\widetilde{1} + \widetilde{\alpha}\right)$.Then f_φ has a unique fixed soft point in $SP(\widetilde{X})$.

Proof. Let x_a^0 be an arbitrary soft point in $SP(\widetilde{X})$. We define a soft sequence as follows:

$$f_\varphi\left(x_{a_n}^n\right) = x_{a_{n-1}}^{n-1}, \text{ for } n = 1, 2, ...$$

Taking $x_{a_{n+1}}^{n+1} = x_a$ and $x_{a_{n+2}}^{n+2} = y_b$ in (3.1), we have

$$b_S\left(f_\varphi\left(x_{a_{n+1}}^{n+1}\right), f_\varphi\left(x_{a_{n+2}}^{n+2}\right), \widetilde{t}\right) +$$
$$+\widetilde{\alpha}\max\left\{b_S\left(x_{a_{n+1}}^{n+1}, f_\varphi\left(x_{a_{n+2}}^{n+2}\right), \widetilde{t}\right), b_S\left(x_{a_{n+2}}^{n+2}, f_\varphi\left(x_{a_{n+1}}^{n+1}\right), \widetilde{t}\right)\right\}$$
$$\geq \widetilde{\beta}\frac{b_S\left(x_{a_{n+1}}^{n+1}, f_\varphi\left(x_{a_{n+1}}^{n+1}\right), \widetilde{t}\right)\left[\widetilde{1} + b_s\left(x_{a_{n+2}}^{n+2}, f_\varphi\left(x_{a_{n+2}}^{n+2}\right), \widetilde{t}\right)\right]}{\widetilde{1} + b_s\left(x_{a_{n+1}}^{n+1}, x_{a_{n+2}}^{n+2}, \widetilde{t}\right)} + \widetilde{\gamma} b_s\left(x_{a_{n+1}}^{n+1}, x_{a_{n+2}}^{n+2}, \widetilde{t}\right),$$
$$\Longrightarrow b_S\left(x_{a_n}^n, x_{a_{n+1}}^{n+1}, \widetilde{t}\right) + \widetilde{\alpha}\max\left\{b_S\left(x_{a_{n+1}}^{n+1}, x_{a_{n+1}}^{n+1}, \widetilde{t}\right), b_S\left(x_{a_{n+2}}^{n+2}, x_{a_n}^n, \widetilde{t}\right)\right\}$$
$$\geq \widetilde{\beta}\frac{b_S\left(x_{a_{n+1}}^{n+1}, x_{a_n}^n, \widetilde{t}\right)\left[\widetilde{1} + b_s\left(x_{a_{n+2}}^{n+2}, x_{a_{n+1}}^{n+1}, \widetilde{t}\right)\right]}{\widetilde{1} + b_s\left(x_{a_{n+1}}^{n+1}, x_{a_{n+2}}^{n+2}, \widetilde{t}\right)} + \widetilde{\gamma} b_s\left(x_{a_{n+1}}^{n+1}, x_{a_{n+2}}^{n+2}, \widetilde{t}\right).$$
$$\Longrightarrow b_S\left(x_{a_n}^n, x_{a_{n+1}}^{n+1}, \widetilde{t}\right) + \widetilde{\alpha} b_S\left(x_{a_n}^n, x_{a_{n+2}}^{n+2}, \widetilde{t}\right) \geq$$
$$\geq \widetilde{\beta} b_S\left(x_{a_n}^n, x_{a_{n+1}}^{n+1}, \widetilde{t}\right) + \widetilde{\gamma} b_s\left(x_{a_{n+1}}^{n+1}, x_{a_{n+2}}^{n+2}, \widetilde{t}\right),$$
$$b_S\left(x_{a_n}^n, x_{a_{n+1}}^{n+1}, \widetilde{t}\right) + \widetilde{\alpha}\widetilde{s}\left[b_S\left(x_{a_n}^n, x_{a_{n+1}}^{n+1}, \widetilde{t}\right) + b_S\left(x_{a_{n+1}}^{n+1}, x_{a_{n+2}}^{n+2}, \widetilde{t}\right)\right] \geq$$
$$\geq \widetilde{\beta} b_S\left(x_{a_n}^n, x_{a_{n+1}}^{n+1}, \widetilde{t}\right) + \widetilde{\gamma} b_s\left(x_{a_{n+1}}^{n+1}, x_{a_{n+2}}^{n+2}, \widetilde{t}\right)$$
$$\Longrightarrow \left(\widetilde{1} + \widetilde{\alpha}\widetilde{s} - \widetilde{\beta}\right) b_S\left(x_{a_n}^n, x_{a_{n+1}}^{n+1}, \widetilde{t}\right) \geq \left(\widetilde{\gamma} - \widetilde{\alpha}\widetilde{s}\right) b_s\left(x_{a_{n+1}}^{n+1}, x_{a_{n+2}}^{n+2}, \widetilde{t}\right), \text{ for all } \widetilde{t} > \widetilde{0}.$$

$$\Longrightarrow b_s\left(x^{n+1}_{a_{n+1}}, x^{n+2}_{a_{n+2}}, \widetilde{t}\right) \leq \frac{\left(\widetilde{1}+\widetilde{\alpha}\widetilde{s}-\widetilde{\beta}\right)}{\left(\widetilde{\gamma}-\widetilde{\alpha}\widetilde{s}\right)} b_S\left(x^n_{a_n}, x^{n+1}_{a_{n+1}}, \widetilde{t}\right)$$
$$= \widetilde{\vartheta} b_S\left(x^n_{a_n}, x^{n+1}_{a_{n+1}}, \widetilde{t}\right), \text{ for all } \widetilde{t} > \widetilde{0}.$$

Here $\widetilde{\vartheta} = \frac{\left(\widetilde{1}+\widetilde{\alpha}\widetilde{s}-\widetilde{\beta}\right)}{\left(\widetilde{\gamma}-\widetilde{\alpha}\widetilde{s}\right)} < \frac{\widetilde{1}}{\widetilde{s}}$. By using induction, we have

$$b_s\left(x^{n+1}_{a_{n+1}}, x^{n+2}_{a_{n+2}}, \widetilde{t}\right) \leq \widetilde{\vartheta}^{n+1} b_S\left(x^0_a, x^1_{a_1}, \widetilde{t}\right).$$

From Lemma 2.2, the soft sequence $\left\{x^n_{a_n}\right\}$ is a Cauchy sequence in $\widetilde{X}$. By the completeness of $\left(\widetilde{X}, b_S, \widetilde{s}, E\right)$, there is a soft point $x^*_a \in SP(\widetilde{X})$ such that $x^n_{a_n} \to x^*_a$ as $n \to \infty$. From the continuity of f_φ, we obtain

$$f_\varphi\left(x^*_a\right) = f_\varphi\left(\lim_{n\to\infty} x^n_{a_n}\right) = \lim_{n\to\infty} f_\varphi\left(x^n_{a_n}\right) \lim_{n\to\infty} x^{n-1}_{a_{n-1}} = x^*_a.$$

So the soft point $x^*_a \in SP(\widetilde{X})$ is a fixed soft point of the mapping f_φ.
Now, let y^*_b be another fixed soft point of f_φ. Then

$$b_s\left(f_\varphi\left(x^*_a\right), f_\varphi\left(y^*_b\right), \widetilde{t}\right) + \widetilde{\alpha}\max\left\{b_S\left(x^*_a, f_\varphi\left(y^*_b\right), \widetilde{t}\right), b_S\left(y^*_b, f_\varphi\left(x^*_a\right), \widetilde{t}\right)\right\}$$
$$\geq \widetilde{\beta}\frac{b_S\left(x^*_a, f_\varphi\left(x^*_a\right), \widetilde{t}\right)\left[\widetilde{1}+b_s\left(y^*_b, f_\varphi\left(y^*_b\right), \widetilde{t}\right)\right]}{\widetilde{1}+b_s\left(x^*_a, y^*_b, \widetilde{t}\right)} + \widetilde{\gamma} b_s\left(x^*_a, y^*_b, \widetilde{t}\right).$$

Hence

$$b_s\left(x^*_a, y^*_b, \widetilde{t}\right) + \widetilde{\alpha} b_s\left(x^*_a, y^*_b, \widetilde{t}\right) \geq \widetilde{\gamma} b_s\left(x^*_a, y^*_b, \widetilde{t}\right)$$
$$b_s\left(x^*_a, y^*_b, \widetilde{t}\right) \geq \left(\widetilde{\gamma}-\widetilde{\alpha}\right) b_s\left(x^*_a, y^*_b, \widetilde{t}\right)$$
$$b_s\left(x^*_a, y^*_b, \widetilde{t}\right) \leq \frac{\widetilde{1}}{\left(\widetilde{\gamma}-\widetilde{\alpha}\right)} b_s\left(x^*_a, y^*_b, \widetilde{t}\right).$$

So $b_s\left(x^*_a, y^*_b, \widetilde{t}\right) = \widetilde{0}$. That is $x^*_a = y^*_b$. We obtain that the fixed soft point of f_φ is unique.

Theorem 3.2 *Let* $\left(\widetilde{X}, b_S, \widetilde{s}, E\right)$ *be a complete parametric soft* $b-$*metric space and* $f_\varphi : \left(\widetilde{X}, b_S, \widetilde{s}, E\right) \to \left(\widetilde{X}, b_S, \widetilde{s}, E\right)$ *be a surjective mapping satisfying the condition (3.1)* x_a, $y_b \in SP(\widetilde{X})$ *and for all* $\widetilde{t} > \widetilde{0}$, *where* $\widetilde{\alpha}, \widetilde{\beta}, \widetilde{\gamma} \geq \widetilde{0}$ *are soft real constants and* $\widetilde{s}\widetilde{\beta} + \widetilde{\gamma} > \left(\widetilde{1}+\widetilde{\alpha}\right)\widetilde{s} + \widetilde{s}^2\widetilde{\alpha}$, $\widetilde{\gamma} > \left(\widetilde{1}+\widetilde{\alpha}\right)$. *Then* f_φ *has a unique fixed soft point in* $SP(\widetilde{X})$.

Proof. Let x^0_a be an arbitrary soft point in $SP(\widetilde{X})$. We define a soft sequence $\left\{x^n_{a_n}\right\}$ in $SP(\widetilde{X})$ as follows:

$$f_\varphi\left(x^n_{a_n}\right) = x^{n-1}_{a_{n-1}}, \text{ for } n = 1, 2, ...$$

By using (3.1), we have Cauchy sequence $\left\{x^n_{a_n}\right\}$ in $SP(\widetilde{X})$. By the completeness of $\left(\widetilde{X}, b_S, \widetilde{s}, E\right)$,there is a soft point $x^*_a \in SP(\widetilde{X})$ such that $x^n_{a_n} \to x^*_a$ as $n \to \infty$.

Now we show that there is a fixed soft poi int in $SP(\widetilde{X})$. Since $f_{\varphi}:\left(\widetilde{X},b_S,\widetilde{s},E\right)\rightarrow\left(\widetilde{X},b_S,\widetilde{s},E\right)$ is a surjective mapping, there is a soft point y_b in $SP(\widetilde{X})$ such that $f_{\varphi}(y_b)=x_a^*$. Then

$$b_S\left(x_{a_n}^n,x_a^*\widetilde{t}\right)=b_S\left(f_{\varphi}\left(x_{a_{n+1}}^{n+1}\right),f_{\varphi}(y_b),\widetilde{t}\right)$$
$$\geq-\widetilde{\alpha}\max\left\{b_S\left(x_{a_{n+1}}^{n+1},f_{\varphi}(y_b),\widetilde{t}\right),b_S\left(y_b,f_{\varphi}\left(x_{a_{n+1}}^{n+1}\right),\widetilde{t}\right)\right\}+$$
$$+\widetilde{\beta}\frac{b_S\left(x_{a_{n+1}}^{n+1},f_{\varphi}\left(x_{a_{n+1}}^{n+1}\right),\widetilde{t}\right)\left[\widetilde{1}+b_s\left(y_b,f_{\varphi}(y_b),\widetilde{t}\right)\right]}{\widetilde{1}+b_s\left(x_{a_{n+1}}^{n+1},y_b,\widetilde{t}\right)}+\widetilde{\gamma}b_s\left(x_{a_{n+1}}^{n+1},y_b,\widetilde{t}\right).$$

For $n\rightarrow\infty$, we obtain

$$\begin{aligned}b_S\left(x_a^*,x_a^*,\widetilde{t}\right) &\geq -\widetilde{\alpha}\max\left\{b_S\left(x_a^*,x_a^*,\widetilde{t}\right),b_S\left(y_b,x_a^*,\widetilde{t}\right)\right\}+\\ &\quad \widetilde{\beta}\frac{b_S\left(x_a^*,x_a^*,\widetilde{t}\right)\left[\widetilde{1}+b_s\left(y_b,x_a^*,\widetilde{t}\right)\right]}{\widetilde{1}+b_s\left(x_a^*,y_b,\widetilde{t}\right)}+\widetilde{\gamma}b_s\left(x_a^*,y_b,\widetilde{t}\right)\\ \widetilde{0} &\geq -\widetilde{\alpha}b_S\left(x_a^*,y_b,\widetilde{t}\right)+\widetilde{\gamma}b_s\left(x_a^*,y_b,\widetilde{t}\right)\\ &\Longrightarrow(\widetilde{\gamma}-\widetilde{\alpha})\,b_S\left(x_a^*,y_b,\widetilde{t}\right)\leq\widetilde{0}\\ &\Longrightarrow b_S\left(x_a^*,y_b,\widetilde{t}\right)=\widetilde{0}\text{ as }\widetilde{\gamma}>\widetilde{\alpha}.\end{aligned}$$

So $x_a^*=y_b$ and $f_{\varphi}(x_a^*)=x_a^*$, f_{φ} has a fixed soft point in $SP(\widetilde{X})$.
Now, let y_b^* be another fixed soft point of f_{φ}. Then

$$b_s\left(f_{\varphi}(x_a^*),f_{\varphi}(y_b^*),\widetilde{t}\right)+\widetilde{\alpha}\max\left\{b_S\left(x_a^*,f_{\varphi}(y_b^*),\widetilde{t}\right),b_S\left(y_b^*,f_{\varphi}(x_a^*),\widetilde{t}\right)\right\}$$
$$\geq\widetilde{\beta}\frac{b_S\left(x_a^*,f_{\varphi}(x_a^*),\widetilde{t}\right)\left[\widetilde{1}+b_s\left(y_b^*,f_{\varphi}(y_b^*),\widetilde{t}\right)\right]}{\widetilde{1}+b_s\left(x_a^*,y_b^*,\widetilde{t}\right)}+\widetilde{\gamma}b_s\left(x_a^*,y_b^*,\widetilde{t}\right).$$

This implies that

$$b_s\left(x_a^*,y_b^*,\widetilde{t}\right)+\widetilde{\alpha}b_s\left(x_a^*,y_b^*,\widetilde{t}\right)\geq\widetilde{\gamma}b_s\left(x_a^*,y_b^*,\widetilde{t}\right).$$

Hence

$$\begin{aligned}b_s\left(x_a^*,y_b^*,\widetilde{t}\right) &\geq (\widetilde{\gamma}-\widetilde{\alpha})\,b_s\left(x_a^*,y_b^*,\widetilde{t}\right)\\ &\Longrightarrow b_s\left(x_a^*,y_b^*,\widetilde{t}\right)\leq\frac{\widetilde{1}}{(\widetilde{\gamma}-\widetilde{\alpha})}b_s\left(x_a^*,y_b^*,\widetilde{t}\right).\end{aligned}$$

So $b_s\left(x_a^*,y_b^*,\widetilde{t}\right)=\widetilde{0}$. That is $x_a^*=y_b^*$. We obtain that the fixed soft point of f_{φ} is unique.

Theorem 3.3 *Let $f_{\varphi},g_{\psi}:\left(\widetilde{X},b_S,\widetilde{s},E\right)\rightarrow\left(\widetilde{X},b_S,\widetilde{s},E\right)$ be two surjective mappings of a complete parametric soft $b-$metric space $\left(\widetilde{X},b_S,\widetilde{s},E\right)$. Consider that f_{φ},g_{ψ} satisfying the following conditions:*

$$b_S\left(f_{\varphi}(g_{\psi}(x_a)),g_{\psi}(x_a),\widetilde{t}\right)+\widetilde{k}b_S\left(f_{\varphi}(g_{\psi}(x_a)),x_a,\widetilde{t}\right)\geq\widetilde{\alpha}b_S\left(g_{\psi}(x_a),x_a,\widetilde{t}\right)$$

and

$$b_S\left(g_{\psi}(f_{\varphi}(x_a)),f_{\varphi}(x_a),\widetilde{t}\right)+\widetilde{k}b_S\left(g_{\psi}(f_{\varphi}(x_a)),x_a,\widetilde{t}\right)\geq\widetilde{\beta}b_S\left(f_{\varphi}(x_a),x_a,\widetilde{t}\right)$$

for all $x_a \in SP(\widetilde{X})$, for all $\widetilde{t} > \widetilde{0}$ and $\widetilde{\alpha}, \widetilde{\beta}, \widetilde{k} \in \mathbb{R}(E)^*$ with $\widetilde{\alpha} > \widetilde{s}\left(\widetilde{1}+\widetilde{k}\right) + \widetilde{s}^2\widetilde{k}$ and $\widetilde{\beta} > \widetilde{s}\left(\widetilde{1}+\widetilde{k}\right) + \widetilde{s}^2\widetilde{k}$. If f_φ or g_ψ is soft continuous mapping, then f_φ and g_ψ also have a common fixed soft point.

Proof. Let x_a^0 be an arbitrary soft point in $SP(\widetilde{X})$. Since f_φ is a surjective mapping, there is $x_{a_1}^1 \in SP(\widetilde{X})$ such that $f_\varphi\left(x_{a_1}^1\right) = x_a^0$. Similarly, since g_ψ is a surjective mapping, there is $x_{a_2}^2 \in SP(\widetilde{X})$ such that $g_\psi\left(x_{a_2}^2\right) = x_{a_1}^1$. By using this process, we construct a soft sequence $\left\{x_{a_n}^n\right\}$ in $SP(\widetilde{X})$ such that

$$x_{a_{2n}}^{2n} = f_\varphi\left(x_{a_{2n+1}}^{2n+1}\right), x_{a_{2n+1}}^{2n+1} = g_\psi\left(x_{a_{2n+2}}^{2n+2}\right),$$

for all $n \in \mathbb{N} \cup \{0\}$. Thus we have

$$b_S\left(f_\varphi\left(g_\psi\left(x_{a_{2n+2}}^{2n+2}\right)\right), g_\psi\left(x_{a_{2n+2}}^{2n+2}\right), \widetilde{t}\right) + \widetilde{k}b_S\left(f_\varphi\left(g_\psi\left(x_{a_{2n+2}}^{2n+2}\right)\right), x_{a_{2n+2}}^{2n+2}, \widetilde{t}\right) \geq$$

$$\geq \widetilde{\alpha}b_S\left(g_\psi\left(x_{a_{2n+2}}^{2n+2}\right), x_{a_{2n+2}}^{2n+2}, \widetilde{t}\right).$$

So, we obtain

$$b_S\left(x_{a_{2n}}^{2n}, x_{a_{2n+1}}^{2n+1}, \widetilde{t}\right) + \widetilde{k}b_S\left(x_{a_{2n}}^{2n}, x_{a_{2n+2}}^{2n+2}, \widetilde{t}\right) \geq \widetilde{\alpha}b_S\left(x_{a_{2n+1}}^{2n+1}, x_{a_{2n+2}}^{2n+2}, \widetilde{t}\right) \tag{3.2}$$

From the condition of parametric soft $b-$metric, since

$$b_S\left(x_{a_{2n}}^{2n}, x_{a_{2n+2}}^{2n+2}, \widetilde{t}\right) \leq \widetilde{s}\left[b_S\left(x_{a_{2n}}^{2n}, x_{a_{2n+1}}^{2n+1}, \widetilde{t}\right) + b_S\left(x_{a_{2n+1}}^{2n+1}, x_{a_{2n+2}}^{2n+2}, \widetilde{t}\right)\right],$$

we have from (3.2)

$$b_S\left(x_{a_{2n+1}}^{2n+1}, x_{a_{2n+2}}^{2n+2}, \widetilde{t}\right) \leq \frac{\widetilde{1}+\widetilde{s}\widetilde{k}}{\widetilde{\alpha}-\widetilde{s}\widetilde{k}} b_S\left(x_{a_{2n}}^{2n}, x_{a_{2n+1}}^{2n+1}, \widetilde{t}\right). \tag{3.3}$$

Also,

$$b_S\left(g_\psi\left(f_\varphi\left(x_{a_{2n+1}}^{2n+1}\right)\right), f_\varphi\left(x_{a_{2n+1}}^{2n+1}\right), \widetilde{t}\right) + \widetilde{k}b_S\left(g_\psi\left(f_\varphi\left(x_{a_{2n+1}}^{2n+1}\right)\right), x_{a_{2n+1}}^{2n+1}, \widetilde{t}\right) \geq$$

$$\geq \widetilde{\beta}b_S\left(f_\varphi\left(x_{a_{2n+1}}^{2n+1}\right), x_{a_{2n+1}}^{2n+1}, \widetilde{t}\right),$$

we take

$$b_S\left(x_{a_{2n-1}}^{2n-1}, x_{a_{2n}}^{2n}, \widetilde{t}\right) + \widetilde{k}b_S\left(x_{a_{2n-1}}^{2n-1}, x_{a_{2n+1}}^{2n+1}, \widetilde{t}\right) \geq \widetilde{\beta}b_S\left(x_{a_{2n}}^{2n}, x_{a_{2n+1}}^{2n+1}, \widetilde{t}\right). \tag{3.4}$$

From the condition of parametric soft $b-$metric, since

$$b_S\left(x_{a_{2n-1}}^{2n-1}, x_{a_{2n+1}}^{2n+1}, \widetilde{t}\right) \leq \widetilde{s}\left[b_S\left(x_{a_{2n-1}}^{2n-1}, x_{a_{2n}}^{2n}, \widetilde{t}\right) + b_S\left(x_{a_{2n}}^{2n}, x_{a_{2n+1}}^{2n+1}, \widetilde{t}\right)\right],$$

we have from (3.4)

$$b_S\left(x_{a_{2n}}^{2n}, x_{a_{2n+1}}^{2n+1}, \widetilde{t}\right) \leq \frac{\widetilde{1}+\widetilde{s}\widetilde{k}}{\widetilde{\beta}-\widetilde{s}\widetilde{k}} b_S\left(x_{a_{2n-1}}^{2n-1}, x_{a_{2n}}^{2n}, \widetilde{t}\right). \tag{3.5}$$

Let $\widetilde{\vartheta}=\max\left\{\frac{\widetilde{1}+\widetilde{s}\widetilde{k}}{\widetilde{\alpha}-\widetilde{s}\widetilde{k}},\frac{\widetilde{1}+\widetilde{s}\widetilde{k}}{\widetilde{\beta}-\widetilde{s}\widetilde{k}}\right\}$. It is clear that $\widetilde{\vartheta}<\frac{\widetilde{1}}{\widetilde{s}}$. From the (3.3) and (3.5), we obtain for all $n\in\mathbb{N}\cup\{0\}$ and $\widetilde{t}>\widetilde{0}$,

$$b_S\left(x_{a_n}^n,x_{a_{n+1}}^{n+1},\widetilde{t}\right)\leq\widetilde{\vartheta}b_S\left(x_{a_{n-1}}^{n-1},x_{a_n}^n,\widetilde{t}\right).$$

From Lemma 2.2, the soft sequence $\left\{x_{a_n}^n\right\}$ is a Cauchy sequence in $\widetilde{X}$. By the completeness of $\left(\widetilde{X},b_S,\widetilde{s},E\right)$,there is a soft point $x_a^*\in SP(\widetilde{X})$ such that $x_{a_n}^n\to x_a^*$ as $n\to\infty$. Thus $x_{a_{2n+1}}^{2n+1}\to x_a^*$ and $x_{a_{2n+2}}^{2n+2}\to x_a^*$ as $n\to\infty$. Without disturbing the generality,we consider that f_φ is a soft continuous mapping. Thus

$$f_\varphi\left(x_{a_{2n+1}}^{2n+1}\right)\to f_\varphi\left(x_a^*\right),n\to\infty.$$

Also $f_\varphi\left(x_{a_{2n+1}}^{2n+1}\right)=x_{a_{2n}}^{2n}\to x_a^*,n\to\infty$. Hence it is obvious that $f_\varphi\left(x_a^*\right)=x_a^*$.

Since g_ψ is a surjective mapping, there is a soft point $z_b\in SP(\widetilde{X})$ such that $g_\psi\left(z_b\right)=x_a^*$. Then

$$b_S\left(f_\varphi\left(g_\psi\left(z_b\right)\right),g_\psi\left(z_b\right),\widetilde{t}\right)+\widetilde{k}b_S\left(f_\varphi\left(g_\psi\left(z_b\right)\right),z_b,\widetilde{t}\right)\geq\widetilde{\alpha}b_S\left(g_\psi\left(z_b\right),z_b,\widetilde{t}\right)$$

implies that

$$\widetilde{k}b_S\left(x_a^*,z_b,\widetilde{t}\right)\geq\widetilde{\alpha}b_S\left(x_a^*,z_b,\widetilde{t}\right).$$

So

$$b_S\left(x_a^*,z_b,\widetilde{t}\right)\leq\frac{\widetilde{k}}{\widetilde{\alpha}}b_S\left(x_a^*,z_b,\widetilde{t}\right).$$

Since $\widetilde{k}<\widetilde{\alpha}$, $b_S\left(x_a^*,z_b,\widetilde{t}\right)=\widetilde{0}$. Hence $x_a^*=z_b$. So $f_\varphi\left(x_a^*\right)=g_\psi\left(x_a^*\right)=x_a^*$. It is clear that x_a^* is a common fixed soft point both of f_φ and g_ψ.

Corollary 3.1 *Let $f_\varphi,g_\psi:\left(\widetilde{X},b_S,\widetilde{s},E\right)\to\left(\widetilde{X},b_S,\widetilde{s},E\right)$ be two surjective mappings of a complete parametric soft $b-$metric space $\left(\widetilde{X},b_S,\widetilde{s},E\right)$.Consider that f_φ,g_ψ satisfying the following conditions:*

$$b_S\left(f_\varphi\left(g_\psi\left(x_a\right)\right),g_\psi\left(x_a\right),\widetilde{t}\right)+\widetilde{k}b_S\left(f_\varphi\left(g_\psi\left(x_a\right)\right),x_a,\widetilde{t}\right)\geq\widetilde{\alpha}b_S\left(g_\psi\left(x_a\right),x_a,\widetilde{t}\right)$$

and

$$b_S\left(g_\psi\left(f_\varphi\left(x_a\right)\right),f_\varphi\left(x_a\right),\widetilde{t}\right)+\widetilde{k}b_S\left(g_\psi\left(f_\varphi\left(x_a\right)\right),x_a,\widetilde{t}\right)\geq\widetilde{\alpha}b_S\left(f_\varphi\left(x_a\right),x_a,\widetilde{t}\right)$$

for all $x_a\in SP(\widetilde{X})$, for all $\widetilde{t}>\widetilde{0}$ and $\widetilde{\alpha},\widetilde{k}\in\mathbb{R}(E)^*$ with $\widetilde{\alpha}>\widetilde{s}\left(\widetilde{1}+\widetilde{k}\right)+\widetilde{s}^2\widetilde{k}$. If f_φ or g_ψ is soft continuous mapping, then f_φ and g_ψ also have a common fixed soft point.

Proof. If we take $\widetilde{\alpha}=\widetilde{\beta}$ above theorem, the proof is obtained.

Corollary 3.2 *Let $f_\varphi:\left(\widetilde{X},b_S,\widetilde{s},E\right)\to\left(\widetilde{X},b_S,\widetilde{s},E\right)$ be a surjective mapping of a complete parametric soft $b-$metric space $\left(\widetilde{X},b_S,\widetilde{s},E\right)$. Consider that f_φ satisfying the following condition:*

$$b_S\left(f_\varphi\left(f_\varphi\left(x_a\right)\right),f_\varphi\left(x_a\right),\widetilde{t}\right)+\widetilde{k}b_S\left(f_\varphi\left(f_\varphi\left(x_a\right)\right),x_a,\widetilde{t}\right)\geq\widetilde{\alpha}b_S\left(f_\varphi\left(x_a\right),x_a,\widetilde{t}\right)$$

for all $x_a\in SP(\widetilde{X})$, for all $\widetilde{t}>\widetilde{0}$ and $\widetilde{\alpha},\widetilde{k}\in\mathbb{R}(E)^*$ with $\widetilde{\alpha}>\widetilde{s}\left(\widetilde{1}+\widetilde{k}\right)+\widetilde{s}^2\widetilde{k}$. If f_φ is soft continuous mapping, then f_φ also has a common fixed soft point.

Proof. If we take $f_\varphi = g_\psi$ above corollary, the proof is obtained.

Conclusion

We have introduced namely parametric soft $b-$metric space which is based on soft point of soft sets. Later we give some kind of interesting fixed point theorems and demonstrate that when continuous and surjective mappings defined on a complete parametric soft $b-$metric space has a unique fixed soft point.

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